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# CENTRALIZERS OF ELEMENTARY ABELIAN $p$-SUBGROUPS AND MOD-$p$ COHOMOLOGY OF PROFINITE GROUPS

HANS-WERNER HENN

## 1. Introduction

*1.1.* Let $G$ be a profinite group and $p$ be a fixed prime. In this paper we will be concerned with $H_c^*(G;\mathbb{F}_p)$, the continuous cohomology of $G$ with coefficients in the trivial module $\mathbb{F}_p$. We will abbreviate $H_c^*(G;\mathbb{F}_p)$ by $H^*(G;\mathbb{F}_p)$, or simply by $H^*G$ if $p$ is understood from the context. We recall that if $G$ is the (inverse) limit of finite groups $G_i$, then $H^*G = \operatorname{colim} H^*G_i$.

Throughout this paper we will assume that $H^*G$ is finitely generated as an $\mathbb{F}_p$-algebra. By work of Lazard [La], it is known that this holds for many interesting groups, for example, for profinite $p$-adic analytic groups like $GL(n,\mathbb{Z}_p)$, the general linear groups over the $p$-adic integers. In case $H^*G$ is finitely generated as an $\mathbb{F}_p$-algebra, Quillen has shown [Q1] that there are only finitely many conjugacy classes of elementary abelian $p$-subgroups of $G$ (i.e., groups isomorphic to $(\mathbb{Z}/p)^n$ for some natural number $n$). In other words, the following category $\mathscr{A}(G)$ is equivalent to a finite category: objects of $\mathscr{A}(G)$ are all elementary abelian $p$-subgroups of $G$; if $E_1$ and $E_2$ are elementary abelian $p$-subgroups of $G$, then the set of morphisms from $E_1$ to $E_2$ in $\mathscr{A}(G)$ consists precisely of those homomorphisms $\alpha : E_1 \to E_2$ of abelian groups for which there exists an element $g \in G$ with $\alpha(e) = geg^{-1}$ for all $e$ in $E_1$. The category $\mathscr{A}(G)$ plays an important role both in Quillen's results and in the work presented here.

This category entered into Quillen's work as follows. The assignment $E \mapsto H^*E$ extends to a functor from the opposite category $\mathscr{A}(G)^{op}$ to graded $\mathbb{F}_p$-algebras and the restriction homomorphisms $H^*G \to H^*E$ (for $E$ running through the elementary abelian $p$-subgroups of $G$) induce a canonical map of algebras $q : H^*G \to \lim_{\mathscr{A}(G)^{op}} H^*E$.

THEOREM 1.2 [Q1]. *Let $G$ be a profinite group and assume that $H^*G$ is a finitely generated $\mathbb{F}_p$-algebra. Then the canonical map $q : H^*G \to \lim_{\mathscr{A}(G)^{op}} H^*E$ is an $F$-isomorphism; in other words, $q$ has the following properties.*

- *If $x \in \operatorname{Ker} q$, then $x$ is nilpotent.*
- *If $y \in \lim_{\mathscr{A}(G)^{op}} H^*E$, then there exists an integer $n$ with $y^{p^n} \in \operatorname{Im} q$.*

*1.3.* In our main result we use the full subcategory $\mathscr{A}_*(G)$ of $\mathscr{A}(G)$ whose objects are all elementary abelian $p$-subgroups except the trivial subgroup. The

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