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DISCRETE SERIES CHARACTERS AND THE LEFSCHETZ FORMULA FOR HECKE OPERATORS

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This paper consists of three independent but related parts. In the first part (§§1–6), we give a combinatorial formula for the constants appearing in the "numerators" of characters of stable discrete series representations of real groups (see §3), as well as an analogous formula for individual discrete series representations (see §6). Moreover, we give an explicit formula (Theorem 5.1) for certain stable virtual characters on real groups; by Theorem 5.2, these include the stable discrete series characters, and thus we recover the results of §3 in a more natural way.

In the second part (§7), we use the character formula given in Theorem 5.1 to rewrite the Lefschetz formula of [GM1] (for the local contribution at a single fixed-point component to the trace of a Hecke operator on weighted cohomology) in the same spirit as that of Arthur's Lefschetz formula [A1] in terms of stable virtual characters on real groups (see Theorem 7.14.B). We then sum the contributions of the various fixed-point components and show that, in the case of middle-weighted cohomology, the resulting global Lefschetz fixed-point formula agrees with Arthur's Lefschetz formula. This gives a topological proof of Arthur's formula.

The third part of the paper (Appendices A and B) is purely combinatorial. In Appendix A, we develop the combinatorics of convex polyhedral cones on which our results on characters of real groups are based. The same combinatorics is used in Appendix B to prove a generalization of a combinatorial lemma of Langlands.

The formula for stable discrete series constants given in Theorem 3.1 is redundant, since it follows easily from Theorems 5.1 and 5.2. Nevertheless, the proof of Theorem 3.1 is instructive and should probably not be skipped by readers interested in the case of individual discrete series constants. Theorem 3.2 is not redundant, and in fact provides the link between our results on stable discrete series constants and individual discrete series constants. (We return to this point later in the introduction.) Because of the redundancy built into the paper, the reader who is mainly interested in the Lefschetz formula only needs to read §5, §7, and a little bit of Appendix A.

Let G be a connected reductive group over $\mathbb{Q}$, and let A_G denote the maximal $\mathbb{Q}$-split torus in the center of G. Let K_G be a maximal compact subgroup of

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