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EDGE-OF-THE-WEDGE TYPE THEOREMS FOR HYPERFUNCTION SOLUTIONS

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To Professor Hikosaburo Komatsu, in celebration of his 60th birthday

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1. Introduction. Let us consider a real analytic manifold M and its closed submanifold N of codimension $d \geqslant 1$. Denote by $Y \subset X$ a complexification of $N \subset M$. Let $\mathscr{M}$ be a coherent module over the sheaf of ring $\mathscr{D}_X$ of holomorphic differential operators on X, and assume that Y is noncharacteristic for $\mathscr{M}$. The main problem that we treat in this article is to find the sufficient conditions on $\mathscr{M}$ for the vanishing of the local cohomologies of the hyperfunction solution complex of $\mathscr{M}$:

$$H^j \mu_N \mathrm{R}\mathscr{H}om_{\mathscr{D}_X}(\mathscr{M}, \mathscr{B}_M) \simeq 0 \quad \text{for} \quad j < d, \tag{1.1}$$

where μ_N denotes Sato's microlocalization functor along N. This problem is very important because many classical results in analysis are deduced from such vanishing theorems on cohomologies. Let us give some examples.

(a) When the codimension d of N is equal to one and N is defined by $\{x_1 = 0\}$ in M, equation (1.1) entails $\Gamma_{M_\pm}\ \mathscr{H}om_{\mathscr{D}_X}(\mathscr{M}, \mathscr{B}_M)|_N \simeq 0$ for $M_\pm = \{\pm x_1 \geqslant 0\} \subset M$. This formula is Holmgren's theorem, which ensures the uniqueness of the solution of initial or boundary value problems.

(b) Let M be $\mathbb{C}^n$ and N its real part $\mathbb{R}^n$. If we take $\mathscr{M} = \bar{\partial}$ as the Cauchy-Riemann system, equation (1.1) is the abstract edge-of-the-wedge theorem of Martineau [19] and Kashiwara [8]:

$$H^j \mu_{\mathbb{R}^n}(\mathscr{O}_{\mathbb{C}^n}) = 0 \quad \text{for} \quad j < d = n. \tag{1.2}$$

This theorem implies various extension theorems of holomorphic functions along $N = \mathbb{R}^n$, and it is essentially used to construct the theory of micro-

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