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HIGHLY RAMIFIED PENCILS OF ELLIPTIC CURVES IN CHARACTERISTIC 2

ERNST-ULRICH GEKELER

The conductor $f = f(E/L)$ of an elliptic curve E over the complete discretely valued field L is a nonnegative integer that measures the complexity of E related to reduction. As is well known, $f = 0$, 1, $\geqslant 2$ if and only if E has good, multiplicative, additive reduction, respectively, where $f > 2$ can only occur if the residue characteristic of L equals 2 or 3. Moreover, if $\mathrm{char}(L) = 0$, the possible values of f are bounded by a constant depending only on the field L [16], [1]. This does not remain true, however, if $\mathrm{char}(L)$ equals 2 or 3.

In the present article, we investigate the possible conductors and ramification types when $\mathrm{char}(L) = 2$ and the residue class field is finite, i.e., $L = \mathbb{F}_q((T))$ and $q = 2^e$. Since L admits separable quadratic extensions L_χ of arbitrarily large conductors $f(L_\chi/L)$, it is quite plausible that the corresponding twists $E \rightsquigarrow E_\chi$ produce arbitrarily large conductors $f(E_\chi/L)$, too. Precise statements are given in Section 1, notably Theorem 1.4, which describes the behavior of f under twists. However, there are less obvious reasons for the unboundedness of $f(E/L)$ when E varies. In Section 4, we construct certain curves $E_{\alpha,\beta}$ (motivated from the global theory of Section 3), whose conductors f take on all the values $f \geqslant 3$, $f \not\equiv 2 \pmod 4$. Some conductor $f(E_{\alpha,\beta})$ is minimal with respect to twists of $E_{\alpha,\beta}$ if and only if it is odd; hence each odd f appears even as a minimal conductor (Proposition 4.4).

In the other sections, we study the ramification of globally defined pencils, i.e., elliptic curves E/K, where K is a rational function field $\mathbb{F}_q(T)$. The case of one ($\mathbb{F}_q$-rational) place v of bad reduction leads to $j(E) \in \mathbb{F}_q$ and is not further pursued (although the curve with $j(E) = 0$ gives rise to interesting constructions; see [7]).

If $j(E)$ is to be nonconstant, one has to admit at least two places v and w of bad reduction. We investigate the case where E has multiplicative reduction at v and additive reduction at w, where without restriction, $v = \infty$, $w = 0$, and E/K_∞ is even a Tate curve. We obtain a complete classification of these curves (Theorem 3.2, Corollary 3.6) together with their conductors (which are unbounded; see Proposition 4.4 and Corollary 4.6) and their isogenies (Theorem 5.5; here we have to assume $q = 2$).

Theorem 1.4 on the conductor $f(E_\chi)$ of the twisted curve E_χ is shown by translating it (by means of the local Langlands correspondence) into a question of representation theory. The main ingredient in the proof of Theorem 3.2 is the

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