Vol. 87, No. 1 DUKE MATHEMATICAL JOURNAL © 1997

ON THE SHORT-TIME BEHAVIOR OF THE FREE BOUNDARY OF A POROUS MEDIUM EQUATION

CARMEN CORTÁZAR, MANUEL DEL PINO, AND MANUEL ELGUETA

1. Introduction and preliminaries. Let Ω be a domain in $\mathbf{R}^N$, $N \geqslant 2$, with a bounded, smooth boundary and $m > 1$. In this paper, we study the following initial-boundary value problem for the porous medium equation,

$$\begin{aligned} u_t &= \Delta u^m \quad \text{in } \Omega \times (0, +\infty) \\ u(x,t) &= 1 \quad \text{in } \partial\Omega \times (0, +\infty), \qquad (1.1) \\ u(x,0) &= 0 \quad \text{in } \Omega. \end{aligned}$$

Problem (1.1) models the flow of a gas into a porous container shaped as Ω. Initially, the container is empty, and then the density of the gas, represented by u, is kept constant and equal to one at the boundary.

An important feature of the solution to (1.1), whose existence and uniqueness is guaranteed by the standard theory for the porous medium equation, is that it propagates with finite speed. By this we mean that if x_0 is an interior point of Ω, then there exists a positive time $T(x_0)$ such that $u(x_0, t) = 0$ for $0 < t < T(x_0)$ and $u(x_0, t) > 0$ for $T(x_0) < t$. See [A].

A natural question is that of estimating the value of $T(x_0)$. Of course, this quantity may depend strongly on the geometry of the domain, and it would be hard to provide a general precise estimate for it. This question was considered in [CE], where Neumann rather than Dirichlet boundary conditions were imposed, and a general upper estimate for $T(x_0)$ was derived in the case when Ω is bounded and convex. The estimate in [CE], involving certain integral quantities depending globally on the domain and the boundary condition, is however not sharp.

Our purpose in this paper is to find a precise estimate for $T(x_0)$ when the point x_0 lies sufficiently close to the boundary of Ω. When this is the case, it is natural to expect an answer that depends only on the local geometry of Ω near x_0.

Received 14 October 1995. Revision received 12 June 1996.

Cortázar and Elgueta partially supported by Fondo Nacional de Ciencia y Technologia (FONDECYT) grant number 1940705.

del Pino partially supported by FONDECYT grant number 1950303 and European Community grant number CI1*-CT93-0323.