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AN ALGORITHM OF COMPUTING b-FUNCTIONS

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To Professor Hikosaburo Komatsu on the occasion of his sixtieth birthday

1. Introduction. Let $f(x) \in K[x] = K[x_1, \ldots, x_n]$ be a polynomial of n variables with coefficients in a field K of characteristic zero. Let us denote by

$$A_n(K) := K[x_1, \ldots, x_n]\langle \partial_1, \ldots, \partial_n \rangle, \qquad \hat{\mathscr{D}}_n(K) := K[[x_1, \ldots, x_n]]\langle \partial_1, \ldots, \partial_n \rangle$$

the rings of differential operators with polynomial and formal power series coefficients, respectively, with $\partial_i = \partial/\partial x_i$ and $\partial = (\partial_1, \ldots, \partial_n)$. ($A_n(K)$ is called the Weyl algebra over K.)

Let s be a parameter. Then the (local) b-function (or the Bernstein-Sato polynomial) $b_f(s)$ associated with $f(x)$ is the monic polynomial of the least degree $b(s) \in K[s]$ satisfying

$$P(s, x, \partial) f(x)^{s+1} = b(s) f(x)^s \tag{1.1}$$

with some $P(s, x, \partial) \in \hat{\mathscr{D}}_n(K)[s]$. The monic polynomial of the least degree $b(s) \in K[s]$ satisfying (1.1) with some $P(s, x, \partial) \in A_n(K)[s]$ is denoted by $\tilde{b}_f(s)$. The existence of $\tilde{b}_f(s)$ was proved by I. N. Bernstein [Be1], [Be2], which implies the existence of $b_f(s)$. Note that $b_f(s)$ divides $\tilde{b}_f(s)$, but $b_f(s)$ and $\tilde{b}_f(s)$ are not necessarily identical. More generally, the existence of $b_f(s)$ for $f(x) \in K[[x]]$ was proved by J. E. Björk [Bj].

In this paper, we present an algorithm for, given $f(x) \in K[x]$, computing $b_f(s)$ and finding a $P(s, x, \partial) \in \hat{\mathscr{D}}_n(K)$ that satisfies (1.1) with $b(s) = b_f(s)$. More precisely, our algorithm finds a $Q(s, x, \partial) \in A_n(K)[s]$ and an $a(x) \in K[x]$ with $a(0) \neq 0$ such that $P(s, x, \partial) = (1/a(x)) Q(s, x, \partial)$ satisfies (1.1) with $b(s) = b_f(s)$. Computing $\tilde{b}_f(s)$ and an associated $P \in A_n(K)[s]$ is slightly easier.

An algorithm of computing $b_f(s)$ was first given by M. Sato et al. [SKKO] when $f(x)$ is a relative invariant of a prehomogeneous vector space. J. Briançon et al. [BGMM] and Ph. Maisonobe [Mai] gave an algorithm of computing $b_f(s)$ for $f(x)$ with isolated singularity. Also note that T. Yano [Y] worked out many interesting examples of b-functions systematically.

Our method consists in computing the (generalized) b-function for a section of a holonomic system (or more generally, a specializable D-module) via Gröbner basis computation in the Weyl algebra. In general, let M be a finitely generated

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