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# ON THE GROWTH OF HIGH SOBOLEV NORMS OF SOLUTIONS FOR *KdV* AND SCHRÖDINGER EQUATIONS

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**1. Introduction.** The aim of this paper is to study the growth of high Sobolev norms of solutions of certain dispersive differential equations.

Here we consider the initial value problems (IVP)

$$\begin{cases} i\partial_t u + \partial_x^2 u + \lambda|u|^k u = 0 \\ u(x,0) = \Phi(x) \qquad x \in \mathbb{R} \text{ or } \mathbb{T},\ t \in \mathbb{R} \end{cases} \tag{1}$$

(nonlinear Schrödinger IVP) and

$$\begin{cases} \partial_t u + \partial_x^3 u + \lambda\partial_x u^k = 0 \\ u(x,0) = \Phi(x) \qquad x \in \mathbb{R}, \text{ or } \mathbb{T},\ t \in \mathbb{R}. \end{cases} \tag{2}$$

(generalized *KdV* IVP).

If in (1) $k = 2$ and $\lambda \in \mathbb{R}$, we have an integrable equation in the sense that there are infinitely many conserved quantities controlling the flow (see [16]). In particular if $u$ is a solution, then for any $s \geqslant 1$

$$\|u(t)\|_{H^s} \leqslant C. \tag{3}$$

The same is true for the IVP (2) when $k = 2, 3$ (see [15] and [14]). On the other hand, if we perturb (1) by replacing the real constant $\lambda$ by a real smooth function $\lambda(x)$, or if we assume $k > 2$, we break the integrability of the equation, and (3) is not known anymore. Similarly, if in (2) we assume that $\lambda$ is a real function, or if $k > 3$, the uniform bound (3) for the solution of the equation is not known.

In general what one can prove, whenever a global solution $u$ of (1) and (2) exists, is that

$$\|u(t)\|_{H^s} \leqslant C^{|t|}, \tag{4}$$

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