Vol. 85, No. 3 DUKE MATHEMATICAL JOURNAL © December 1996

ON THE HITCHIN SYSTEM

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1. Introduction

1.1. N. J. Hitchin in his 1987 paper [H2] introduced a new class of algebraically completely integrable (ACI) hamiltonian systems. The motion is linear on the Prym variety of a spectral curve, which was also introduced by Hitchin [H1], while the phase space is the cotangent bundle to the moduli space of stable bundles over a fixed Riemann surface. This idea gave great impetus to the study of moduli spaces of vector bundles over a curve through the "abelianization program," which makes use of covering curves and their abelian varieties (cf. [BNR]). Hitchin himself considered more general systems, and many modifications have ensued; the most comprehensive set of references to date can be found in [DM]. In this paper we only consider the case of rank-2 vector bundles with trivial determinant, over a Riemann surface C of genus $g > 1$. In that case the Hitchin system is obtained as follows. Let

$$\mathscr{M} := \{E \to C\colon E \text{ a semistable rank-2 bundle, } \wedge^2 E \cong \mathscr{O}\}/\sim_S$$

be the moduli space of (S-equivalence classes of) semistable rank-2 vector bundles over C. $\mathscr{M}$ is a projective variety of dimension $3g - 3$. The locus of stable bundles $\mathscr{M}^s$ is the set of smooth points of $\mathscr{M}$ for $g > 2$. The cotangent space of $\mathscr{M}$ at a stable bundle E is

$$T_E^*\mathscr{M} = \mathrm{Hom}_0(E, E \otimes K), \qquad \text{with} \quad \mathrm{Hom}_0(E, E \otimes K) := H^0(C, \mathscr{E}nd_0(E) \otimes K)$$

where $\mathscr{E}nd_0(E)$ is the sheaf of endomorphisms of E with trace zero, and K is the canonical bundle of C. An element $\Phi \in \mathrm{Hom}_0(E, E \otimes K)$ is called a Higgs field. The determinant of a Higgs field $\det(\Phi) \in \mathrm{Hom}(\wedge^2 E, \wedge^2(E \otimes K)) = H^0(C, 2K)$ gives a map

$$\det\colon T_E^*\mathscr{M} = \mathrm{Hom}_0(E, E \otimes K) \to H^0(C, 2K),$$

which globalizes to a map on $T^*\mathscr{M}^s$. Hitchin considered the map

$$H\colon T^*\mathscr{M}^s \to H^0(C, 2K), \qquad \Phi \mapsto \det(\Phi)$$

Received 13 July 1995.