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ABELIAN C*-SUBALGEBRAS OF C*-ALGEBRAS OF REAL RANK ZERO AND INDUCTIVE LIMIT C*-ALGEBRAS

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§0. Summary of main results. Let $X = S^{n_1} \times S^{n_2} \times \cdots \times S^{n_k}$, or let X be an absolute retract. It is shown that if a monomorphism $\phi\colon C(X) \to A$ is homotopically trivial, then ϕ can be approximated pointwise by homomorphisms from $C(X)$ into A with finite-dimensional range, provided that A belongs to a certain class of simple C*-algebras of real rank zero. These C*-algebras include all purely infinite simple C*-algebras, the Bunce-Deddens algebras, and the irrational rotation algebras. It is also shown (as a consequence) that if A is a simple C*-algebra which is the inductive limit of a sequence of C*-algebras of the form

$$C(X_k) \otimes M_{n_k},$$

with each X_k a contractible compact metric space, and if A is assumed to have real rank zero and only countably many extreme traces, then A is an AF-algebra.

§1. Introduction. A C*-algebra is said to have real rank zero if the set of selfadjoint elements with finite spectrum is dense in A_{sa}, the set of all selfadjoint elements of A. C*-algebras of real rank zero have recently been under rather intense study (see [BP], [Ell1], [Ell2], [EE], [BBEK], [BDR], [Zh1]–[Zh6], [Lin1]–[Lin8], [LZ], [GL], [Ph1]–[Ph2], etc.). The above-mentioned definition for C*-algebras of real rank zero involves an abelian C*-subalgebra which is isomorphic to $C(X)$, where X is a compact subset of the real line. One may wonder whether there is an analogous property for a general abelian C*-subalgebra of a C*-algebra of real rank zero. In fact, the analogue for $C(S^1)$ is the following: a unital C*-algebra has real rank zero if and only if the set of unitaries with finite spectrum is dense in the connected component of the unitary group containing the identity (see [Lin5]).

It was shown in [Lin8] that for the purely infinite simple C*-algebras, the Bunce-Deddens algebras, the irrational rotation algebras, and many other simple C*-algebras of real rank zero, the following holds: a normal element x can be approximated by a normal element with finite spectrum if and only if a certain index, $\Gamma(x)$, is zero. (This says that the K_1-class associated to each hole in the spectrum of x is zero.)

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