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RESTRICTION THEOREMS AND SEMILINEAR KLEIN-GORDON EQUATIONS IN (1 + 3)-DIMENSIONS

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1. Introduction. The main goal of this paper is to prove global and "almost global" existence theorems for certain semilinear Klein-Gordon equations. Specifically, if

$$(1.1)\qquad \begin{cases} |\partial^j F_p(u)| \leqslant C|u|^{p-j}, & 0 \leqslant j \leqslant [p] \\ |F_p'(u) - F_p'(v)| \leqslant C|u-v|^{p-j}, & \text{if } 1 < p < 2, \end{cases}$$

with $[p]$ denoting the greatest integer $\leqslant p$, then we shall study existence problems in $\mathbb{R}_+^{1+n} = \mathbb{R}_+ \times \mathbb{R}^n$ for

$$(1.2)\qquad \begin{cases} \Box u + u = F_p(u) \\ u(0,x) = f(x), \qquad \partial_t u(0,x) = g(x), \end{cases}$$

if $\Box = \partial_t^2 - \Delta$ denotes the d'Alembertian. We shall assume that the data involved is small, compactly supported, and sufficiently smooth.

In the linear case where $F_p(u) = 0$, it is well known that the solution u is $O((1+t)^{-n/2})$ as $t \to \infty$. If this decay also occurs in the nonlinear case, then one would be able to apply the energy inequality and these pointwise bounds to estimate the H^1 Sobolev norm of u with bounds independent of t precisely when $(p-1)n/2 > 1$. In other words, this heuristic argument suggests that (1.2) should always have a global solution for small C_0^∞ data if

$$(1.3)\qquad p > 1 + 2/n.$$

We should point out that this argument leads to the incorrect prediction for nonlinear wave equations $\Box u = F_p(u)$. There, a solution of the linear equation decays like $t^{-(n-1)/2}$ at infinity, and hence the above considerations might lead one to suspect that the nonlinear equations should admit global solutions when $p > 1 + 2/(n-1)$. John [6], though, showed that this is not the case. Indeed, when $n = 3$, the above argument predicts global existence for semilinear wave equations when $p > 2$, but John showed that there can be blowup for arbitrarily

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