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CORRELATION FOR SURFACES OF GENERAL TYPE

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1. Introduction. The purpose of this paper is to prove the following theorem.

THEOREM 1.1 (Correlation theorem for surfaces). *Let $f: X \to B$ be a proper morphism of integral varieties, whose general fiber is an integral surface of general type. Then for n sufficiently large, X^n_B admits a dominant rational map h to a variety W of general type such that the restriction of h to a general fiber of f^n is generically finite.*

This theorem has a number of geometric and number-theoretic consequences which will be discussed in the final section of this paper. In particular, assuming Lang's conjecture on rational points of varieties of general type, we prove a uniform bound on the number of rational points on a surface of general type that are not contained in rational or elliptic curves.

The inspiration for this paper is the work of Caporaso, Harris, and Mazur [CHM], where the correlation theorem is proved for families of curves of genus $g \geqslant 2$. The same result is conjectured for families of varieties of general type of any dimension. The paper [CHM] contains many of the ideas needed for a proof of the general conjecture. However, at one point the argument hinges on the fact that the fibers of the map are curves: it invokes the existence of a "nice" class of singular curves, the stable curves. For the purpose of this discussion, "nice" means two things.

1. Given any proper morphism $f: X \to B$ whose generic fiber is a smooth curve of genus $g \geqslant 2$, there exists a generically finite base change $B' \to B$ so that the dominating component $X' \subset X \times_B B'$ is birational to a family of stable curves over B'.
2. Let $X \to B$ be a family of stable curves, smooth over the generic point. Then the fiber products X^n_B have canonical singularities.

For the purpose of generalizing to higher dimensions, we make the following definitions.

Let $\mathscr{C}$ be a class of singular varieties.

$\mathscr{C}$ is *inclusive* if for any proper morphism $f: X \to B$ whose generic fiber is a variety of general type, there is a generically finite base change $B' \to B$ so that the dominating component $X' \subset X \times_B B'$ is birational to a family $Y' \to B'$ with fibers in $\mathscr{C}$.

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