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ON THE FUNCTORS CW_A AND P_A

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1. Introduction. Let A be a pointed and connected space. A pair of spaces (Y, X) is called a relative A-CW-complex if, roughly speaking, Y can be obtained from X by wedging with suspensions of A and attaching cones on suspensions of A (see [6, Corollary 3.7]). If $A = S^1$, then a relative S^1-CW-complex is essentially an ordinary relative CW-complex. Any pointed map $f: X \to Y$ can be factored as a composition $(X \to Y' \xrightarrow{p} Y)$, where (Y', X) is a relative A-CW-complex and p induces a weak equivalence of mapping spaces $p_*: map_*(A, Y') \to map_*(A, Y)$.

Let X be a pointed space. By factoring $* \to X$, we get a map $CW_AX \to X$, where $(CW_AX, *)$ is a relative A-CW-complex and the induced $map_*(A, CW_AX) \to map_*(A, X)$ is a weak equivalence. The assignment $X \mapsto CW_AX$ can be made functorial, in such a way that the map $CW_AX \to X$ is natural.

By factoring $X \to *$, we get a map $X \to P_AX$, where (P_AX, X) is a relative A-CW-complex and the space $map_*(A, P_AX)$ is weakly contractible. The assignment $X \mapsto P_AX$ can be made functorial, in such a way that the map $X \to P_AX$ is natural.

The functors CW_A and P_A are crucial in studying spaces through the "eyes" of A. The functor CW_A assigns to a space X the largest subobject $CW_AX \to X$, which is totally "visible" by A, while the functor P_A associates with X the largest quotient $X \to P_AX$, which is totally "invisible" by A. The space CW_AX contains all the information about X that can be detected by A, while P_AX contains all the information about X that cannot be detected by A at all.

The purpose of this paper is to study the relationship between the functors CW_A and P_A. We study these functors by looking at their images and kernels. The image of CW_A (respectively, of P_A) is the class of all spaces X, for which there exists Y, such that X is weakly equivalent to CW_AY (respectively, X is weakly equivalent to P_AY). The kernel of CW_A (respectively, of P_A) is the class of all spaces X, for which CW_AX is weakly contractible (respectively, P_AX is weakly contractible).

We investigate to what extent the following sequence is "exact":

$$\cdots \xrightarrow{P_A} cSpaces_* \xrightarrow{CW_A} cSpaces_* \xrightarrow{P_A} cSpaces_* \xrightarrow{CW_A} cSpaces_* \xrightarrow{P_A} \cdots$$

where $cSpaces_*$ is the category of pointed and connected spaces.

As the first result, we prove that the image of P_A coincides with the kernel of

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