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CONTINUITY OF RELATIVE HYPERBOLIC SPECTRAL THEORY THROUGH METRIC DEGENERATION

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§0. Introduction and background material. Let M denote a Riemann surface of signature (g, n); hence M can be realized as a compact Riemann surface of genus g with n points removed. A metric on M is determined by a positive (1, 1) form μ. All metrics on M are assumed to be compatible with the complex structure on the underlying compact algebraic curve M'. Associated to the metric μ is a positive Laplacian, which we denote by $\Delta_{\mu,M}$. In a local coordinate $z = x + iy$ on M, if the metric μ is given by

$$\mu(z) = \rho^{-1}(z)\frac{i}{2}\, dz \wedge d\bar{z}, \tag{0.1}$$

then the Laplacian $\Delta_{\mu,M}$ is given by

$$\Delta_{\mu,M} = -\rho(z)\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) = -4\rho(z)\left(\frac{\partial^2}{\partial z\, \partial \bar{z}}\right). \tag{0.2}$$

The first Chern form $c_1(\mu) = c_1(\rho)$ of the metric μ is the (1, 1) defined locally by

$$c_1(\mu) = dd^c \log \rho$$

where

$$dd^c = \frac{\sqrt{-1}}{2\pi}\partial\bar{\partial} = \frac{\sqrt{-1}}{2\pi}\frac{\partial^2}{\partial z\, \partial \bar{z}}\, dz \wedge d\bar{z}.$$

The associated Griffiths function $G(\mu) = G(\rho)$ is the function defined by

$$G(\mu)\mu = c_1(\mu).$$

Classically, $-G$ is the Gauss curvature of the metric μ (see page 100 of [La]).

Assume for now that $n = 0$, so M is a compact Riemann surface of genus g. Since M is compact, it is classical that the action of the Laplacian $\Delta_{\mu,M}$ on the space of smooth functions has a discrete spectrum with positive eigenvalues. The

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