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RELATIONS AMONG DONALDSON INVARIANTS ARISING FROM NEGATIVE 2-SPHERES AND TORI

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1. Introduction. The basic problem in 4-manifold theory of determining the minimal genus of a surface representing a given 2-dimensional homology class has been attacked recently with great success using the tools of gauge theory. The strongest results to date are due to Kronheimer and Mrowka [KM1], who prove that surfaces of *positive* self-intersection in a 4-manifold M impose strong restrictions on the Donaldson polynomials of M. These results yield, on the one hand, calculations of the Donaldson polynomials of large classes of manifolds, and on the other hand, the determination of the minimal genus of positive homology classes in these manifolds.[1] The purpose of this paper is to describe relations among the Donaldson polynomials of 4-manifolds containing 2-spheres and tori of *negative* self-intersection. The method is a combination of those used in [MMR] to deal with positive 2-spheres and tori, together with some additional work to account for reducible connections. The relations yield an obstruction to representing a homology class of square $-1, -2$, and -3 by an embedded torus.

To describe the results, let us employ the following notation: M will denote a smooth, oriented manifold with an orientation of $H_2^+(M)$, in short a *homology orientation.* A capital letter A or C will denote either a smooth surface in M or the homology class it carries. The Donaldson polynomial invariant of a manifold M with b_2^+ odd and greater than 1 will be denoted in the case of SU(2) bundles by D (or D_M for clarity) so that the energy k ($= c_2$ of the relevant bundle) can be deduced from the number of classes on which the polynomial is evaluated. For SO(3) bundles for which an integral lift of w_2 of the bundle is Poincaré dual to A, we will use the notation D^A for the Donaldson polynomial, where now $k = -1/4p_1$. The choice of integral lift $PD(A)$ for w_2 determines the orientation of the moduli space as described in [D1]. For an evaluation of D on a class C which is repeated r times and an unspecified collection $\Sigma = \{\Sigma_1, \dots, \Sigma_s\}$, we will use the notation $D(C^r, \Sigma)$. The following theorem is the basic result; in §5 we will use elementary arguments to deduce from it results about embedded 2-spheres, and about tori of square -1 and 0.

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[1] Recent results [KM3], [MST], [R] using the Seiberg-Witten equations reproduce these results and extend them to manifolds with $b_2^+ = 1$.