MODULAR MOONSHINE II

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1. Introduction. The monster simple group acts on the monster vertex algebra, and the moonshine conjectures state that the traces of elements of the monster on the vertex algebra are hauptmoduls. Ryba [R94] conjectured the existence of similar vertex algebras over fields of characteristics p acted on by the centralizers of certain elements of prime order p in the monster, and he conjectured that the Brauer traces of p-regular elements of the centralizers were certain hauptmoduls. We will prove these conjectures when the centralizer involves a sporadic group ($p \leqslant 11$, corresponding to the sporadic groups B, Fi'_{24}, Th, HN, He, and M_{12}).

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The original "moonshine conjectures" of Conway, Norton, McKay, and Thompson said that the monster simple group M has an infinite-dimensional graded representation $V = \bigoplus_{n \in \mathbf{Z}} V_n$ such that the dimension of V_n is the coefficient of q^n of the elliptic modular function $j(\tau) - 744 = q^{-1} + 196884q + \cdots$, and, more generally, that the McKay-Thompson series $T_g(\tau) = \sum_{n \in \mathbf{Z}} \operatorname{Tr}(g \,|\, V_n) q^n$ is a hauptmodul for some genus-0 congruence subgroup of $SL_2(\mathbf{R})$. The representation V was constructed by Frenkel, Lepowsky, and Meurman [FLM], and it was shown to satisfy the moonshine conjectures in [B92] by using the fact that it carries the structure of a vertex algebra [B86], [FLM].

Meanwhile, Norton had suggested that there should be a graded space associated to every element g of the monster acted on by some central extension of the centralizer of g ([N]; see also [Q]). It is easy to see that these graded spaces are usually unlikely to have a vertex algebra structure. Ryba suggested [R94] that these spaces might have a vertex algebra structure if they were reduced mod