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# NEW DUAL PAIR CORRESPONDENCES

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**Introduction.** Let $\mathfrak{g}$ be an exceptional complex Lie algebra of type $\mathbf{F}_4$, $\mathbf{E}_6$, $\mathbf{E}_7$, or $\mathbf{E}_8$. Then $\mathfrak{g}$ has a unique real form $\mathfrak{g}_0$ with real rank four (see [OV], pages 315–316). Let $G$ be the simply connected algebraic group defined over $\mathbb{R}$ such that the Lie algebra of $G(\mathbb{R})$ is $\mathfrak{g}_0$. These four groups form a family indexed by the four alternative real algebras $\mathbb{R}$, $\mathbb{C}$, $\mathbb{H}$, and $\mathbb{O}$ of dimensions $z = 1, 2, 4$, and 8. Gross and Wallach [GW] constructed a minimal unitary representation $\tilde{V}$ of $G(\mathbb{R})$ (or the twofold cover $\tilde{G}(\mathbb{R})$ in the case $\mathbf{F}_4$). Another construction of the minimal representation $\tilde{V}$ has been announced by Brylinski and Kostant [BK].

Let $Q$ be one of the four alternative algebras. Let $J_Q$ be the real Jordan algebra consisting of hermitian $3 \times 3$ matrices with coefficients in $Q$. Let $H$ be a connected algebraic group defined over $\mathbb{R}$ such that $H(\mathbb{R})$ is the connected component of the automorphism group of $J_Q$. Note that $H(\mathbb{R})$ is compact. Let $\mathbf{G}_2(\mathbb{R})$ be the split real algebraic group of type $\mathbf{G}_2$. Then $\mathbf{G}_2(\mathbb{R}) \times H(\mathbb{R})$ is a dual reductive pair in $G_{ad}(\mathbb{R})$, the quotient of $G(\mathbb{R})$ by its center. In this paper we restrict $\tilde{V}$ to $\mathbf{G}_2(\mathbb{R}) \times H(\mathbb{R})$.

We obtain a decomposition

$$\tilde{V}|_{\mathbf{G}_2(\mathbb{R}) \times H(\mathbb{R})} = \oplus \Theta(E) \otimes E,$$

where the sum is taken over (some) finite-dimensional, irreducible representations $E$ of $H(\mathbb{R})$. We show that $\Theta(E)$ is an irreducible representation of $\mathbf{G}_2(\mathbb{R})$ (or the twofold cover $\tilde{\mathbf{G}}_2(\mathbb{R})$ in the case $\mathbf{F}_4$) and describe it in terms of Vogan's classification [V].

The correspondence $E \leftrightarrow \Theta(E)$ is one-to-one in all cases but one. In the $\mathbf{E}_6$ case, we get that $\Theta(E) \cong \Theta(E^*)$. This, however, has a natural explanation. The Dynkin diagram of type $\mathbf{E}_6$ has an automorphism of order two. The corresponding automorphism of $G(\mathbb{R})$ fixes $\mathbf{G}_2(\mathbb{R})$ and induces an automorphism of $H(\mathbb{R})$ which sends $E$ into $E^*$. A similar result was obtained in [S].

In the cases $\mathbf{E}_n$, ($n = 6, 7, 8$), we have obtained correspondences of representations of algebraic groups, so it is tempting to ask whether they are new examples of the Langlands correspondences. Indeed, for $n = 6, 7$, the groups $\mathbf{G}_2$ and $H$ are of almost equal rank, i.e., their ranks differ at most by one, and we formulate the correspondences in terms of L-packets. For $n = 8$, however, $H$ is much bigger,

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