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NILPOTENCE FOR MODULES OVER THE MOD 2 STEENROD ALGEBRA, II

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1. Introduction. The periodicity theorem of Hopkins and Smith [7] (see also [16]) is an almost formal consequence of the nilpotence theorem of Devinatz, Hopkins, and Smith [4]; given a finite spectrum X, the former result classifies many of the nonnilpotent elements in $[X, X]$, the homotopy classes of self-maps of X. In [11], we prove an analog of the nilpotence theorem for modules over the mod 2 Steenrod algebra A, describing functors which detect nilpotence in $\mathrm{Ext}_A^{**}(M, M)$ for any finite A-module M—see Theorem 2.8 below. In this paper we imitate as much as we can of [7]; our main result is the existence of many nonnilpotent elements in $\mathrm{Ext}_A^{**}(M, M)$ for any finite A-module M. Note that we describe what is probably only a small subset of the collection of nonnilpotent elements in $\mathrm{Ext}_A^{**}(M, M)$, so our result is not as complete as the geometric result in [7]; however, we do succeed in establishing much of the framework used in the geometric setting there.

To detect nilpotence in $\mathrm{Ext}_A^{**}(M, M)$, one restricts to elementary sub-Hopf algebras. In this paper we use these sub-Hopf algebras to define certain kinds of self-maps, analogous to v_n-maps, and we work out some of their properties. We use these properties to prove the existence of certain nonnilpotent elements in $\mathrm{Ext}_A^{**}(M, M)$ for M a finite A-module. Restricting to elementary sub-Hopf algebras is not as nice, algebraically, as computing Morava K-theories, so we are not able to develop things as fully as in [7]. In particular, we do not prove an analog of the thick subcategory theorem.

Our main results are the following. Recall that P_t^s is the Milnor basis element dual to $\xi_t^{2^s}$, and that $(P_t^s)^2 = 0$ if $s < t$; hence for any A-module M, if $s < t$ then P_t^s acts as a differential on M. Roughly speaking, for nonnegative integers $s < t$, an *$h_{t,s}$-map* y in $\mathrm{Ext}_A^{**}(\mathbf{F}_2, \mathbf{F}_2)$ is an element represented by some power of $h_{t,s} = [\xi_t^{2^s}]$ in the cobar complex; alternatively, it is an element that restricts to a power of the generator in $\mathrm{Ext}_{E[P_t^s]}^{**}(\mathbf{F}_2, \mathbf{F}_2) \cong \mathbf{F}_2[h_{t,s}]$—see Definition 3.1 for a precise definition. Note that such a y will have bidegree $(k, 2^s(2^t - 1)k)$ for some k. For example, the element usually written as h_0 or h_{10} in $\mathrm{Ext}_A^{1,1}(\mathbf{F}_2, \mathbf{F}_2)$ is an $h_{1,0}$-map; $g \in \mathrm{Ext}_A^{4,24}(\mathbf{F}_2, \mathbf{F}_2)$ is an $h_{2,1}$-map. These (and their powers) are the only known examples of $h_{t,s}$-maps in $\mathrm{Ext}_A^{**}(\mathbf{F}_2, \mathbf{F}_2)$, but we assert that there are many others.

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