AN INVARIANT FOR YAMABE-TYPE FLOWS WITH APPLICATIONS TO SCALAR-CURVATURE PROBLEMS IN HIGH DIMENSION

ABBAS BAHRI

We introduce, in this paper, an invariant for Yamabe-type flows, and we use it in order to give partial answers to the scalar-curvature problems in dimension $n \geqslant 7$.

Using this method, Theorems 1 and 2 of [1] hold, under some additional hypothesis, essentially technical. We recently announced [2] that the algebraic topology argument of [1] contained a gap.

Trying to remove it, we were led to introduce an algebraic expression, some kind of degree, which had to be nonzero for the proof of [1] to work. This degree is automatically 1 for Yamabe-type problems (when $K \equiv 1$), but not obviously so for scalar-curvature problems.

It turns out that this degree is part of an invariant, for a certain family of pseudogradient flows of scalar-curvature problems, denoted P_1. Under suitable conditions, we derive from this invariant the existence of a solution to the scalar-curvature problem. We could have used this invariant to remove the gap in the algebraic topology argument of [1], but we chose rather to give a direct, shorter proof, in a simplified framework.

Since [1] contains general cancellation principles in variational theory, very delicate analysis at infinity, and useful Morse lemmas, we will publish it—with the necessary correction—despite the mistake in it. It will have more generality than the present paper.

Here we chose to consider a simplified situation, where the topology brought by the function K (see below) to the scalar-curvature variational problem is spherical.

In this simplified framework, the arguments will be more transparent. In papers to come, we will extend this invariant to more general situations, so that the framework of [1] can be handled (i.e., situations where the topology brought in by K is more complicated than spherical). We will also study, in papers to come, various properties and extensions of this invariant, from the algebraic topology point of view.

Let

(1) $$K: S^n \to \mathbb{R}$$

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