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CONVERGENCE THEOREMS FOR RELATIVE SPECTRAL FUNCTIONS ON HYPERBOLIC RIEMANN SURFACES OF FINITE VOLUME

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0. Introduction and notation. Let M denote a *not necessarily connected* Riemann surface of signature (g, n). *It is important that we consider surfaces which are not necessarily connected. Unless explicitly stated, surfaces need not be connected.* The genus g is defined to be the sum of the genera of the components, and the number of cusps n is the sum of the number of cusps on each component. Let $m_0(M)$ be the number of connected components of M.

A metric on M is determined by a smooth, positive (1, 1) form μ_M, and all metrics on M are assumed to be complete and to be compatible with the complex structure on the underlying algebraic curve. Associated to the metric μ_M is a positive Laplacian, which we denote by $\Delta_{\mu,M}$. In a local coordinate $z = x + iy$ on M, one can write the metric μ_M and the corresponding Laplacian $\Delta_{\mu,M}$ as

$$\mu_M(z) = \rho^{-1}(z)\frac{i}{2}dz \wedge d\bar{z} \quad \text{and} \quad \Delta_{\mu,M} = -4\rho(z)\frac{\partial^2}{\partial z \partial \bar{z}} .$$

Assume for now that $n = 0$, so M is compact (again, not necessarily connected). Since M is compact, it is classical that the action of the Laplacian $\Delta_{\mu,M}$ on the space of smooth functions has a discrete spectrum with nonnegative eigenvalues. The multiplicity of the zero eigenvalue is equal to the number of connected components of M. The *nonzero* eigenvalues will be expressed by the sequence

$$0 < \lambda_{1,\mu}(M) \leqslant \lambda_{2,\mu}(M) \leqslant \cdots .$$

Denote the associated set of unit L^2-norm eigenfunctions of $\Delta_{\mu,M}$ by $\{\psi_{n,\mu}(M)\}$, so this set of eigenfunctions forms a complete orthonormal basis for the Hilbert space of L^2 functions on M. Recall that the differential equation satisfied by the eigenfunctions is

$$\Delta_{\mu,M}\psi_{n,\mu}(M) - \lambda_{n,\mu}(M)\psi_{n,\mu}(M) = 0 . \tag{0.1}$$

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