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SYMPLECTIC AND POISSON STRUCTURES OF CERTAIN MODULI SPACES, II: PROJECTIVE REPRESENTATIONS OF COCOMPACT PLANAR DISCRETE GROUPS

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Introduction. Let π be a finitely generated orientation preserving infinite cocompact planar discrete group of euclidean or hyperbolic motions. So π acts by isometries on the euclidean or upper half plane (as appropriate); the orbit space Σ is a compact orientable Riemann surface; and π is the *orbifold fundamental group* of Σ. Compare, e.g., [47, p. 423] and what is said in Section 6 below. In the hyperbolic case, π is also called a (cocompact) *Fuchsian group*. Let G be a Lie group, with an adjoint action invariant nondegenerate symmetric bilinear form on its Lie algebra g, not necessarily positive definite. The set of representations, or, more generally, projective representations, of π in G is known to inherit additional structure, under suitable circumstances; cf., e.g., [6], [19], [26], [27], [30], [31], [33], [36], [38], [41], [44], [48], [49], [50]. In this paper, we shall study the symplectic, or, more generally, *Poisson*, geometry of such representations. More specifically, extending a certain construction carried out in an earlier paper [38] for the fundamental group of a closed surface, we shall obtain certain *extended representation spaces*; see below for a precise definition. In [38], the theory has been made for arbitrary finite presentations, but it has been applied only to the standard presentation of the fundamental group of a closed surface. However, the general approach in [38] applies to an arbitrary group of the kind π and yields the following. See Theorem 2.9 for a more precise statement.

THEOREM. *There is a smooth symplectic manifold $\mathcal{M}$ containing* $\mathrm{Hom}(\pi, G)$ *together with a hamiltonian G-action on $\mathcal{M}$ and a momentum mapping in such a way that the reduced space equals the space* $\mathrm{Rep}(\pi, G)$ *of representations of π in G. More generally, spaces of projective representations of π are obtained by symplectic reduction at appropriate nonzero values of the momentum mapping.*

The (smooth) symplectic manifold $\mathcal{M}$, together with the G-action and momentum mapping, is what we mean by an *extended representation space*. The second clause of the theorem will be made precise in Section 3. Our result, apart from being interesting in its own right, reveals some interesting and attractive geometric properties of these twisted representation spaces, which have been spelled out

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