Vol. 79, No. 2 DUKE MATHEMATICAL JOURNAL © August 1995

A CLASS OF SOLUTIONS FOR THE NEUMANN PROBLEM $-\Delta u + \lambda u = u^{(N+2)/(N-2)}$

MASSIMO GROSSI

Introduction. In this paper we study the problem

$$(0.1) \qquad \begin{cases} -\Delta u + \lambda u = u^{(N+2)/(N-2)} & \text{in } \Omega \\ u > 0 & \text{in } \Omega \\ \partial u/\partial \nu = 0 & \text{on } \partial\Omega, \end{cases}$$

where $N \geqslant 3$, $\lambda > 0$, Ω is a bounded smooth domain of $\mathbb{R}^N$ and ν denotes the outer normal vector to $\partial\Omega$.

As it is well known, the main difficulty in studying (0.1) is that the corresponding variational problem lacks compactness, i.e., the functionals related to (0.1) do not satisfy the Palais-Smale condition. The first existence result for (0.1) in general domains and λ large has been obtained by Adimurthi and Mancini (see [AM1]) and X. J. Wang (see [W1]). More precisely, if we set

$$(0.2) \qquad Q_\lambda(u) = \frac{\int_\Omega (|\nabla u|^2 + \lambda u^2)}{(\int_\Omega (|u|^{2N/(N-2)})^{(N-2)/N}}$$

and

$$(0.3) \qquad S_\lambda = \inf\{Q_\lambda(u), u \in H^1(\Omega)\backslash\{0\}\},$$

[AM] and [W1] prove the following.

THEOREM 0.1. *There exists $\lambda_0 > 0$ such that for all $\lambda > \lambda_0$, problem (0.1) admits a solution u_λ which minimizes Q_λ (i.e., $Q_\lambda(u_\lambda) = S_\lambda$). Moreover this solution satisfies*

$$(0.4) \qquad S_\lambda < \frac{S}{2^{2/N}} ,$$

where S is the best constant for the Sobolev embedding $H_0^1(\Omega) \to L^{2N/(N-2)}(\Omega)$.

Received 10 August 1994.