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# BOUNDING HOMOTOPY AND HOMOLOGY GROUPS BY CURVATURE AND DIAMETER

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**0. Introduction.** In this paper, we are concerned with the set of compact Riemannian $n$-manifolds with uniform bounds on the absolute value of sectional curvature and diameter. Since we can scale the metric, we will always assume that $|K| \leqslant 1$.

Given a positive integer $n \geqslant 2$ and a real number $D > 0$, let $\mathcal{M}_D^n$ denote the set of the isometry classes of the Riemannian $n$-manifolds which satisfy

$$|K| \leqslant 1, \text{diam} \leqslant D.$$

The main results of this paper (Theorems 0.1–0.6) show that many interesting topological invariants of the homotopy groups and homology groups of $M \in \mathcal{M}_D^n$ can be bounded in terms of $n$ and $D$ (with the normalized bound on sectional curvature, $|K| \leqslant 1$).

Let $N(\cdot, \cdot)$ denote a constant whose value depends on parameters in the parenthesis.

THEOREM 0.1. *Given $n \geqslant 2$ and $D > 0$, let $M \in \mathcal{M}_D^n$. For each $q \geqslant 1$, the minimal number of generators for $\pi_q(M)$ is bounded by $N_1(n, D, q) < \infty$, provided $\pi_q(M)$ is finitely generated.*

Since $|\pi_1(M)| < \infty$ implies that for each $q \geqslant 1$, $\pi_q(M)$ is finitely generated (see [Sp]), we immediately get the following.

COROLLARY 0.2. *Given $n \geqslant 2$ and $D > 0$, let $M \in \mathcal{M}_D^n$. Assume that $|\pi_1(M| < \infty$. Then, for each $q \geqslant 1$, the minimal number of generators for $\pi_q(M)$ is bounded by $N_1(n, D, q)$.*

Theorem 0.3 was suggested to the author by K. Grove.

THEOREM 0.3. *Given $n \geqslant 2$ and $D > 0$, let $M \in \mathcal{M}_D^n$. For each $q \geqslant 2$, there are at most $N_2(n, D, q)$ isomorphism classes for the qth rational homotopy group $\pi_q(M) \otimes \mathbb{Q}$.*

COROLLARY 0.4. *Given $n \geqslant 2$ and $D > 0$, let $M \in \mathcal{M}_D^n$. For each $q \geqslant 2$, if* $\operatorname{rank}(\pi_q(M)) < \infty$, *then* $\operatorname{rank}(\pi_q(M)) \leqslant N_2(n, D, q)$.

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