Vol. 78, No. 2 DUKE MATHEMATICAL JOURNAL © May 1995

LOCAL BEHAVIOR OF SINGULAR POSITIVE SOLUTIONS OF SEMILINEAR ELLIPTIC EQUATIONS WITH SOBOLEV EXPONENT

CHIUN-CHUAN CHEN AND CHANG-SHOU LIN

1. Introduction. In this paper we study the local behavior of a positive smooth solution u near the singular set $\mathbf{Z}$ of the conformally invariant scalar curvature equation

$$\Delta u + u^{(n+2)/(n-2)} = 0 \qquad \text{in } \Omega\backslash\mathbf{Z}, \tag{1.1}$$

where Ω is an open domain and $\mathbf{Z}$ is a closed set in $\mathbf{R}^n$. One of the motivations in studying the equation (1.1) arises in the problem of finding a metric which is conformal to the flat metric in $\mathbf{R}^n$ and has a positive constant scalar curvature. The works of Schoen [S1] and Schoen and Yau [SY] on complete locally conformally flat manifolds indicated the importance of studying solutions of (1.1) with a singular set $\mathbf{Z}$. When $\mathbf{Z}$ consists of only one single point, say, the origin, Caffarelli, Gidas, and Spruck [CGS] proved that u is asymptotically symmetric near zero, that is, $u(x) = \bar{u}(|x|)(1 + o(1))$ as $x \to 0$, where $\bar{u}(|x|)$ is the average of u on the sphere $S_{|x|}$. And then they went on to show that u has a precise behavior near zero, i.e.,

$$u(x) = v(x)(1 + o(1)) \tag{1.2}$$

where $v(x) = v(|x|)$ is an entire singular solution of

$$\begin{cases} \Delta v + v^{(n+2)/(n-2)} = 0 & \text{in } \mathbf{R}^n\backslash\{0\}, \\ v > 0, & \lim\limits_{|x|\to 0} v(x) = +\infty. \end{cases} \tag{1.3}$$

In this paper, we prove a weaker version of their result to the general case when the singular set is not isolated. The method developed in [CGS], a combination of the Alexandrov's reflection method and a measure theoretic estimate of admissible directions, is quite complicated, and seems difficult to treat the general case. We may take a different approach to attack this problem. First, we prove an a priori estimate near zero,

$$u(x) \leqslant c|x|^{-(n-2)/2}. \tag{1.4}$$

Received 28 June 1994.