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RESTRICTIONS ON THE GEOMETRY AT INFINITY OF NONNEGATIVELY CURVED MANIFOLDS

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1. Introduction and basic results. One of the most fruitful approaches to the study of open manifolds arises from the understanding of their geometry at infinity. This is, for example, the underlying idea in the proof of Mostow's rigidity theorem, and in many of the results dealing with Hadamard manifolds.

The structure of noncompact manifolds with complete Riemannian metrics of nonnegative curvature became fairly well understood after the work of Cheeger, Gromoll, and Meyer in the seventies. The soul theorem provides a good description of the differentiable structure of such manifolds. Namely, there exists a compact totally geodesic submanifold S, the soul, embeddded in M, whose normal bundle is diffeomorphic to the ambient space M ([CG]; for the differentiability part, check [G] or [Po]).

However, it was only in the eighties that Gromov introduced an analogue of the ideal boundary for this class of manifolds [BGS]. His approach (which is explicitly detailed in Section 3 of this paper) consists in introducing a metric in the space of rays, where we have identified previously those rays that have not grown apart fast enough. In a series of exercises included in the same reference, he outlined some of the main consequences that metric properties of $M(\infty)$ have in M and vice versa.

A further development was carried out by Kasue [K], who provided explicit proofs of most of the statements made by Gromov, together with a natural extension of the definition of ideal boundary to a bigger class of manifolds, namely those with asymptotically nonnegative curvature. In this paper, though, we will not deal with this broader class and will remain within the nonnegative curvature bound. For this case, Shioya provided a very readable introduction to the concept of ideal boundary in his paper [Shi].

A new motivation for the study of this object is its relation to collapsing problems under a lower curvature bound. Given a convergent sequence (under the Gromov-Hausdorff topology) of manifolds of a fixed dimension n with sectional curvatures bounded below, we will say that this sequence collapses if its Gromov-Hausdorff limit has Hausdorff dimension smaller than n. According to [GP], the limit in this case is an Alexandrov space of curvature bounded below. No other restrictions on the limit are known. In particular, it is not known whether or not any Alexandrov space can be obtained in this way.

The collapsing phenomenon occurs naturally when one considers a pointed

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