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ON A PROBLEM OF MOSER

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0. Introduction. This paper studies the analytic structure of the local hull of holomorphy of a 2-dimensional, real analytic manifold that is embedded in $\mathbb{C}^2$. Our specific purpose is to solve a problem of Jurgen Moser (see [Mos], [MoW]). In the statement of this problem we shall use certain standard terminology from the literature that will be defined later.

THEOREM 0.1. *Let M be a 2-dimensional, real analytic embedded submanifold of $\mathbb{C}^2$. Suppose that $z_0 \in M$ is a nondegenerate elliptic point of M. Then the local hull of holomorphy $\widetilde{M}$ of M near z_0 is a Levi flat hypersurface which is real analytic across the boundary manifold M.*

Recall that for a general closed subset $E \subseteq \mathbb{C}^n$, we define here the *hull of holomorphy* $\widetilde{E}$ of E to be the intersection of all Stein neighborhoods of E.

The determination of the hull of holomorphy of a subset $E \subseteq \mathbb{C}^n$ is a fundamental problem in complex analysis. In the 1960s, when it was studied by E. Bishop [Bis], one of the motivations was the study of analytic structure in the maximal ideal space of a Banach algebra. Today, now that the function theory of several complex variables is more developed, there are more basic reasons for studying this problem. The papers [BeKl], [Gr], and [Eli] give several instances of the important role of this circle of ideas in the literature.

In full generality, the aforementioned problem is very difficult. Bishop [Bis] first proposed that the hull of holomorphy be determined by using analytic discs attached to M, in the case when M is a smooth, regularly embedded submanifold of $\mathbb{C}^n$. (It is known in general, however—see [Stolz]—that a set may have a large hull of holomorphy that contains no analytic discs.) He classified the local study of the hull in terms of the local geometry of the base point $z_0 \in M$. Namely, it is now understood that it is important to distinguish the case when the tangent space $T_{z_0}M$ has a complex structure from the case when $T_{z_0}M$ is totally real (that is, $T_{z_0}M \cap \sqrt{-1}T_{z_0}M = \{0\}$). Points of the second type are of no interest for us because, by the work of Hörmander-Wermer [HoW], the local hull of holomorphy near such a point contains no new points. The situation in the first case is quite different.

When M is a 2-dimensional real submanifold in $\mathbb{C}^2$, Bishop [Bis] showed that, in the case that $z_0 \in M$ has an isolated complex tangent and satisfies a certain nondegeneracy condition, then a holomorphic change of variables may be

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