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AUTOMORPHIC INDUCTION FOR $GL(n)$ (OVER LOCAL NONARCHIMEDEAN FIELDS)

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1. Introduction

1.1. Let F be a nonarchimedean locally compact field and $\bar{F}$ a Galois closure of F. If $n \geqslant 2$ is an integer, the Langlands conjecture for $GL(n)$ generalises class field theory, which concerns $n = 1$, by predicting a canonical correspondence between isomorphism classes of degree n continuous complex representations of the Galois group $\mathscr{G}(\bar{F}/F)$ and isomorphism classes of smooth irreducible complex representations of $GL(n, F)$. This should actually become a (canonical) bijection if we replace $\mathscr{G}(\bar{F}/F)$ by the Deligne-Weil group.

The Langlands conjecture is known to hold when F has positive characteristic, but only for $n = 2, 3$ when F has characteristic zero. It nevertheless raises the following question. Take any natural operation on Galois representations of nonarchimedean local fields. How does it translate to smooth irreducible representations of linear groups? If we could get a translation purely in terms of smooth irreducible representations of linear groups, it could even help to establish the Langlands conjecture.

In this paper we want to translate the process of induction of Galois representations.

1.2. Let E be a finite extension of F in $\bar{F}$, and d its degree. Then $\mathscr{G}(\bar{F}/E)$ is an open subgroup of $\mathscr{G}(\bar{F}/F)$, of index d, and we can induce continuous complex representations of $\mathscr{G}(\bar{F}/E)$ to continuous complex representations of $\mathscr{G}(\bar{F}/F)$. This multiplies the degree by d. Consequently, if $m \geqslant 1$ is any integer and $n = md$, we expect that there is a map, which we call local automorphic induction (or by the generic term of lifting since our context is clear) which to each class of smooth irreducible representations τ of $GL(m, E)$ attaches a class of smooth irreducible representations π of $GL(n, F)$. As is the case for base change (the operation on smooth irreducible representations of $GL(n, F)$ corresponding to the restriction to $\mathscr{G}(\bar{F}/E)$ of representations of $\mathscr{G}(\bar{F}/F)$), we cannot expect at present to get results unless E/F is cyclic. So assume this, and let κ: $F^\times \to \mathbf{C}^\times$ be a character defining E. Then, at least when τ is tempered, τ should determine π via an identity relating the character of τ to the κ-twisted character of π; so a priori π may depend upon κ, and we say π is a κ-lift of τ. A κ-lift π should be κ-stable: $(\kappa \circ \det) \otimes \pi \simeq \pi$, since representations of $\mathscr{G}(\bar{F}/F)$ induced from E are characterised by the analogous condition, and a κ-stable π (at least if it is tempered) should be a lift.

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