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RESIDUES AND COHOMOLOGY OF COMPLETE INTERSECTIONS

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1. Introduction. Let V be the transversal intersection of the smooth hypersurfaces H_k: $f_k(x) = 0$, $k = 1, \ldots, c$ in the complex projective space $\mathbb{P}^m$. Let $n = m - c$ denote the dimension of V and let $\deg(f_k) = d_k$ for $k = 1, \ldots, c$ be the degrees of the defining polynomials.

If j_V: $V \to \mathbb{P}^m$ is the inclusion, define the *primitive* cohomology of V to be

$$H_0^*(V) = \text{Coker}\{j_V^*: H^*(\mathbb{P}^m) \to H^*(V)\},$$

where complex coefficients are used for cohomology when not stated otherwise.

Let A denote the quotient-graded ring $\mathbb{C}[x_0, \ldots, x_m]/(f_1, \ldots, f_c)$ and let ω_A be a free A-module with a generator of degree $-d(V)$, where $d(V) = d_1 + \cdots + d_c - m - 1$. Let N be a free-graded A-module of rank c with a basis e_i for $i = 1, \ldots, c$, where $\deg(e_i) = -d_i$. Consider the map

$$t: \text{Der}_{\mathbb{C}}(\mathbb{C}[x_0, \ldots, x_m], A) \to N$$

given by $t(\delta) = \delta(f_1)e_1 + \cdots + \delta(f_c)e_c$. The graded module

$$T(V) = \text{Coker}(t)$$

is the space of first-order infinitesimal deformations of A, the coordinate ring of the affine cone $(CV, 0)$; see, for instance, [Lj]. Let S^iM denote the ith symmetric power of an A-module M and let $H_0^{p,q}(V)$ be the primitive (p, q)-type subspace in $H_0^n(V)$ for $p + q = n$. With this notation, we have the following (unpublished) result due to Buchweitz. For a proof in the general case of quasi smooth complete intersections in weighted projective spaces, we refer to Steenbrink [S, Chapter 1], while closely related facts are in Flenner [F].

THEOREM 1. *There is a perfect pairing*

$$H_0^{n-p,p}(V) \otimes_{\mathbb{C}} (S^pT(V) \otimes_A \omega_A)_0 \to \mathbb{C},$$

where the subscript refers to the degree-zero homogeneous component.

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