Vol. 77, No. 1 DUKE MATHEMATICAL JOURNAL © January 1995

GEOMETRICAL FINITENESS WITH VARIABLE NEGATIVE CURVATURE

B. H. BOWDITCH

0. Introduction. A *Hadamard manifold* is a complete, simply connected Riemannian manifold of nonpositive curvature. By a *pinched Hadamard manifold*, we shall mean a Hadamard manifold of pinched negative curvature; i.e., all the sectional curvatures lie between two negative constants. The aim of this paper is to describe a notion of "geometrical finiteness" for a discrete group, Γ, acting on a pinched Hadamard manifold X.

The notion of geometrical finiteness has been principally used in the case where X is 3-dimensional hyperbolic space $\mathbb{H}^3$. The original definition supposed that Γ should possess a finite-sided fundamental polyhedron. Under this hypothesis, Ahlfors showed that the limit set of Γ has either zero or full spherical Lebesgue measure [Ah]. Since that time, other definitions of geometrical finiteness have been given, notably by Marden [M], Beardon and Maskit [BeM], and Thurston [T], and the notion has become central to the study of 3-dimensional hyperbolic groups.

As an isolated object, a geometrically finite group is not particularly interesting. A major problem in 3-dimensional hyperbolic geometry is to understand finitely generated discrete hyperbolic groups that are not geometrically finite. An important conjecture is that every such group is an "algebraic limit" of geometrically finite groups.

In 3 dimensions, Teichmüller theory together with ideas of Thurston have provided powerful tools for understanding hyperbolic groups. In higher dimensions, the theory is much less well developed, and there has been some confusion in the literature as to the correct notion of geometrical finiteness in this context. The existence of finite-sided fundamental polyhedra, without further qualification, becomes an inapropriate hypothesis. My previous paper [Bo1] was an attempt to clarify this matter.

It seems natural to wonder what happens if one generalises in another direction, by allowing variable curvature. The extra flexibility would potentially allow for more possibilities in the construction of exotic examples. The first step, however, is to clearly understand the "geometrically finite" groups. This paper is aimed in that direction.

Let us suppose that Γ is a discrete group of isometries of a pinched Hadamard manifold, X. We want to say what it means for Γ to be geometrically finite.

Received 16 December 1992. Revision received 20 July 1994.

Article written at Institute des Hautes Études Scientifiques, Bures-sur-Yvette, with the support of a Royal Society European Exchange Fellowship.