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ON THE DIRICHLET PROBLEM FOR HARMONIC MAPS WITH PRESCRIBED SINGULARITIES

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1. Introduction. The Einstein vacuum equations in the stationary axially symmetric case reduce to a harmonic map from $\mathbb{R}^3$ into $\mathbb{H}^2_{\mathbb{R}}$, the hyperbolic plane, with prescribed singularities along the axis of symmetry. In [18] and [19] we used this fact to construct solutions of these equations which could be interpreted as a pair of rotating black holes held apart by a singular strut. These solutions generalized the static Weyl solutions; see [1]. The first step in this program was to solve a Dirichlet problem for such maps with the singularity prescribed along a closed submanifold of the domain. A natural generalization of this problem is to replace the Einstein vacuum equations with the Einstein-Maxwell equations. A similar reduction again leads to a harmonic map problem with prescribed singularities, but the target is now $\mathbb{H}^2_{\mathbb{C}}$, the complex hyperbolic plane; see [11].

In this paper, we study the Dirichlet problem for harmonic maps with prescribed singularities from a smooth bounded domain $\Omega \subset \mathbb{R}^n$, $n \geqslant 2$, into (M, g), a classical Riemannian globally symmetric space of rank one and of noncompact type. Thus (M, g) is either the real-, complex-, or quaternion-hyperbolic space, i.e., $(M, g) = \mathbb{H}^{\ell}_{\mathbb{K}}$, where $\ell \geqslant 2$, and $\mathbb{K}$ is either $\mathbb{R}$, $\mathbb{C}$, or the quaternions $\mathbb{H}$; see [6]. For simplicity, we take the Euclidean metric on $\mathbb{R}^n$, although all the results carry over easily to bounded domains in Riemannian manifolds. Recall that a map $\varphi\colon \Omega \to (M, g)$ is *harmonic* if for each $\Omega' \subset\subset \Omega$ the map $\varphi|\Omega'$ is a critical point of the energy:

$$E_{\Omega'}(\varphi) = \int_{\Omega'} |d\varphi|^2, \tag{1}$$

where $|d\varphi|^2 = \sum_{k=1}^{n} g(\nabla_k \varphi, \nabla_k \varphi)$. It then satisfies an elliptic system of nonlinear partial differential equations, written in local coordinates on M as:

$$\Delta\varphi^a + \sum_{k=1}^{n} \Gamma^a_{bc} \partial_k \varphi^b \partial_k \varphi^c = 0,$$

where Γ^a_{bc} are the Christoffel symbols of (M, g). Harmonic maps have been studied extensively. The Dirichlet problem for harmonic maps into a manifold of non-

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