Vol. 77, No. 1 DUKE MATHEMATICAL JOURNAL © January 1995

THE METHOD OF LAYER POTENTIALS IN ELECTROMAGNETIC SCATTERING THEORY ON NONSMOOTH DOMAINS

MARIUS MITREA

1. Introduction. The two components of an electromagnetic wave (E, H) are related via the Maxwell equations which, in the time-independent case, read

$$(\mathscr{M}) \begin{cases} \operatorname{curl} E - ikH = 0 \\ \operatorname{curl} H + ikE = 0, \end{cases}$$

where $k \neq 0$ is a complex number depending solely on the electric and magnetic characteristics of the medium in $\mathbf{R}^3$ (cf. [6], [7], [11]).

The main purpose of this work is to initiate the study of boundary-value problems for the Maxwell system $(\mathscr{M})$ on arbitrary Lipschitz domains in $\mathbf{R}^3$. In this setting, we obtain existence and uniqueness results, with corresponding optimal estimates, provided the boundary data are in (appropriate subspaces of) $L^p(\partial\Omega)$, for $2 - \varepsilon \leqslant p \leqslant 2 + \varepsilon$. Moreover, the solutions are expressed in terms of layer-potential operators from which interior Sobolev regularity results can be obtained. Finally, we also study the expansions of these solutions in series of electric dipoles a priori distributed.

This program extends to the Lipschitz case the classical treatment of smooth domains initiated in 1954 by Calderón [3]. The problem of determining the optimal range of p's for which this works remains still open.

Since Coifman, McIntosh, and Meyer proved in 1981 the boundedness of the Cauchy operator on arbitrary Lipschitz curves [5], the method of layer potentials has become a very effective alternative for treating boundary-value problems for elliptic and parabolic differential operators, or even systems of such operators on domains with Lipschitz boundaries [32], [8], [9], [10], [14], [1]. (See also the excellent survey papers [18] and [20] for more historical clues.)

The boundary-value problems for the Maxwell equations in smooth domains (i.e., C^2 or $C^{1+\varepsilon}$) were solved for the first time in the early 1950s [3], [26], [33] in terms of singular layer potential integral operators. Due to the smoothness of the boundary of the domain, these integrals are actually only weakly singular, hence giving rise to compact operators which can be readily handled via Fredholm theory. More recently, it has been shown in [25] that the limiting case of domains with only C^1 boundaries is still directly treatable by means of Fredholm theory.

Received 25 August 1993. Revision received 22 June 1994.
The author was supported in part by O.N.R.