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KAZHDAN-LUSZTIG CONJECTURE FOR AFFINE LIE ALGEBRAS WITH NEGATIVE LEVEL

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To the memory of our friend Ansgar Schnizer who died in Kyoto in a tragic car accident

0. Introduction. A fundamental problem in the theory of highest-weight modules over Kac-Moody Lie algebras is to determine the characters of the irreducible highest-weight modules. When the highest weight is dominant integral, the answer for finite-dimensional semisimple Lie algebras is given by classical Weyl's character formula, and its generalization to symmetrizable Kac-Moody Lie algebras is due to V. Kac. However, it is much more difficult to deal with general highest weights, and there still remain unsolved cases.

This problem was initiated by Verma, and then in pursuing this problem for finite-dimensional semisimple Lie algebras, several important results were obtained in the 1970s by Bernstein-Gelfand-Gelfand, Jantzen et al., using algebraic methods. A remarkable breakthrough in this problem was made in 1979 by Kazhdan-Lusztig [KL1]. They gave a conjectural character formula in the case of finite-dimensional semisimple Lie algebras, and suggested that the formula is closely related to the intersection cohomologies of the Schubert varieties. This conjecture was independently solved by Beilinson-Bernstein [BB] and Brylinski-Kashiwara [BK] using the D-module theory. Modules over the Lie algebra correspond to D-modules on the flag manifold, and D-modules correspond to perverse sheaves by the Riemann-Hilbert correspondence. Intersection cohomologies naturally enter in this picture since the intersection cohomology sheaf of a Schubert variety corresponds to an irreducible highest-weight module. This correspondence between modules over Lie algebra and D-modules on the flag manifold established a new approach to the representation theory of semisimple Lie algebras.

The next problem was to generalize it to Kac-Moody Lie aglebras. It is possible to formulate two natural generalizations, one concerned with dominant weights and the other with antidominant weights. Those two cases give different problems because dominant weights and antidominant weights are not conjugate under the action of the Weyl group.

The first dominant case was treated by Kashiwara (-Tanisaki) [K2], [KT] and independently by Casian [C1], where the formula was proved for any symmetrizable Kac-Moody Lie algebra. The method of the proof is similar to the original finite-dimensional case except for the point that special care is taken in deal-

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