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ON THE PRIME IDEAL THEOREM AND IRREGULARITIES IN THE DISTRIBUTION OF PRIMES

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1. Introduction. For any fixed $k \in \mathbb{N}$, let $f(x)$ denote the polynomial $x^k + Qd$ where $Q, d \in \mathbb{N}$. Write, for fixed Q and prime p,

$$\varrho(d, p) = |\{m(\text{mod } p) \colon m^k + Qd \equiv 0 \ (\text{mod } p)\}|.$$

Since the zeros of $f(x)$ are distinct, we can factorize f over $\mathbb{Q}$ as

$$f(x) = \prod_{i=1}^{\nu(d)} f_i(x)$$

where each $f_i(x)$ is irreducible and has integral coefficients.

For fixed k, d and Q, the prime ideal theorem implies that

$$\sum_{p \leqslant x} \varrho(d, p) = \nu(d)\ell i(x) + O\left(\frac{x}{\log^A x}\right), \qquad x \to \infty$$

where A is an arbitrary positive constant and the implied constant depends on k, A, d and Q. The dependence on d and Q, in particular, tends to be of a quality that rules out the possibility of averaging over d in an interval and yet obtaining the expected error term with good uniformity in Q.

In this paper, we prove by a combination of various methods the following result.

THEOREM 1. *Let $k \in \mathbb{N}$, $X, Y \geqslant 2$, $\varepsilon, A > 0$ and $Y^{(1/2)+\varepsilon} \leqslant H \leqslant Y$. Then*

$$\sum_{Y \leqslant d \leqslant Y+H} \left| \sum_{p=X}^{2X} (\varrho(d, p) - \nu(d)) \right| \ll_{A,k,\varepsilon} \frac{XH}{\log^A X}$$

uniformly for $Q \leqslant Y^A$.

The implicit constant in the upper bound is ineffective owing to the application of the Siegel-Brauer theorem in the proof. It is worthwhile to point out that the absence of any restriction in the ranges for X and Y is primarly due to the possibility of interpreting both p and d as a modulus in an appropriate sense.

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