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LOCAL REGULARITY OF SOLUTIONS TO WAVE EQUATIONS WITH TIME-DEPENDENT POTENTIALS

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To the memory of M. Carmen Gazólaz

1. Introduction. In this paper we continue the investigation we started in [RV] concerning regularizing properties of Schrödinger flows. We shall focus our attention mainly on two aspects: first, trying to relax the conditions on the growth of the potential up to be able to consider singularities of the type $c(1/|x|^2)$ with a small $|c|$ (notice that this smallness condition is a natural restriction—see [RS, p. 172]), and second, to analyse the corresponding properties for the hyperbolic wave equation allowing perturbations with potentials as those in the Schrödinger setting.

We shall proceed as in [RV], obtaining suitable Sobolev-type estimates for the corresponding constant coefficient operators to find an inverse written as a Neumann series. Let us start by considering the initial value problems

$$(1.1)\qquad \begin{cases} L_1 u_1(x,t) = F(x,t), & (x,t) \in \mathbb{R}^n \times \mathbb{R}, \\ u(x,0) = 0, \end{cases}$$

where L_1 denotes the time-dependent Schrödinger operator $(1/ih)\partial_t + \Delta_x$, where h is Planck's constant, and

$$(1.2)\qquad \begin{cases} L_2 u_2(x,t) = F(x,t), & (x,t) \in \mathbb{R}^n \times \mathbb{R}, \\ u_2(x,0) = 0 \\ \partial_t u_2(x,0) = 0, \end{cases}$$

for L_2 the Wave operator $\lambda^{-2}\partial_{tt} - \Delta_x$.

We need also to clarify the way we measure the singularities of the potentials. Define the Morrey-Campanato class $\mathscr{L}^{\alpha,p}$, $1 \leqslant p \leqslant n/\alpha$, $\alpha > 0$ as

$$(1.3)\qquad \mathscr{L}^{\alpha,p} = \Big\{ V \in L^p_{\mathrm{loc}}(\mathbb{R}^n), \text{ such that } \|V\|_{\mathscr{L}^{\alpha,p}} \equiv \sup_{r,x_0} \left(r^\alpha \left(r^{-n} \int_{B(x_0,r)} |V(x)|^p \, dx \right)^{1/p} \right) < \infty \Big\}.$$

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