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ANALYTIC DISCS AND THE REGULARITY OF CR MAPPINGS IN HIGHER CODIMENSION

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Introduction. Let M and $\tilde{M}$ be real smooth manifolds in $\mathbf{C}^N$ and $\mathbf{C}^{\tilde{N}}$ respectively. Is any CR mapping between them also smooth? The case of hypersurfaces being understood fairly well (see, e.g., [F1]), we consider the case of higher codimension here.

Let M be a real smooth manifold in $\mathbf{C}^N$. We denote by $T_p^c(M)$ the maximal complex subspace of the tangent space $T_p(M)$ at the point $p \in M$. Recall that the manifold M is called *generic* if $T_p(M) + JT_p(M) = T_p(\mathbf{C}^N) \simeq \mathbf{C}^N$, $p \in M$, where J is the operator of multiplication by the imaginary unit in $\mathbf{C}^N$. A generic manifold M is always a *CR manifold*, which means that all spaces $T_p^c(M)$, $p \in M$, have the same dimension. This dimension is called the *CR dimension* of M and denoted by CRdim(M) here. Recall also that a smooth complex-valued function (mapping) on M is called a *CR function* (*mapping*) if its differential is $\mathbf{C}$-linear on $T^c(M)$.

We denote by $C^{k,\alpha}(k \geqslant 0, 0 < \alpha < 1)$ the class of functions whose derivatives through order k satisfy a Lipschitz condition with exponent α. By a wedge with the edge M, we mean a set of the form $(M + C) \cap U$, where C is an open cone in $\mathbf{C}^N$ and U is a neighborhood of M. The substance of this paper is the following.

THEOREM. *Let $M \subset \mathbf{C}^N$ and $\tilde{M} \subset \mathbf{C}^{\tilde{N}}$ be $C^{k,\alpha}(k \geqslant 2, 0 < \alpha < 1)$ smooth generic submanifolds of positive CR dimensions and $f\colon M \to \tilde{M}$ a C^1 smooth CR mapping such that:*

(a) *f extends holomorphically into a wedge with the edge M;*

(b) *for every point $p \in M$, the differential f_* maps $T_p^c(M)$ onto $T_{\tilde{p}}^c(\tilde{M})$, $\tilde{p} = f(p)$;*

(c) *the Levi form of $\tilde{M}$ is nondegenerate.*

Then f is $C^{k-1,\beta}$ for any β such that $0 < \beta < \alpha$.

Although the theorem holds in the totally real case (CRdim($\tilde{M}$) = 0) as well, we intentionally exclude it from the statement because stronger results hold in this special case. In particular, one need not assume the initial smoothness C^1, and the mapping f turns out to be as smooth as M and $\tilde{M}$ (see [ABR], [F1], [PH]).

Simple examples show that all the conditions (a) through (c) in the theorem are relevant. Indeed, if (a) is omitted, we can take $M = \tilde{M} = M_0 \times \mathbf{R} \subset \mathbf{C}^{N-1} \times \mathbf{C}$ and $f(z, t) = (z, g(t))$, where M_0 is generic in $\mathbf{C}^{N-1}$ and g is C^1 but not smoother. Instead of (a), we can require that M be *minimal* in the sense of [T1], which ensures wedge extendibility of CR functions whence CR mappings.

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