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DEFORMATIONS OF PICARD SHEAVES AND MODULI OF PAIRS

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0. Introduction. Let C be a smooth projective curve of genus $g \geqslant 3$ and let $M(2, \xi)$ denote the moduli space of stable vector bundles V of rank 2 and $\det V \cong \xi$. For $d := \deg \xi$ odd, let $\mathscr{U}_\xi$ denote the universal bundle on $C \times M_\xi$. The Picard sheaf $\mathscr{W}_\xi$ on M_ξ is defined as

$$\mathscr{W}_\xi := p_*(\mathscr{U}_\xi)$$

where $p: C \times M_\xi \to M_\xi$ is the canonical projection. Note that for $d \gg 0$, $\mathscr{W}_\xi$ is locally free.

Our aim in this paper is to study the variations of the Picard bundle $\mathscr{W}_\xi$ on the moduli space M_ξ for $\deg \xi \gg 0$ ($d \geqslant 4g - 2$, to be precise) and $\deg \xi$ odd. A detailed investigation of the variations and the cohomology of the Picard bundle on the Jacobian of the curve was done by G. Kempf in a foundational paper [K]. In our study we make extensive use of the constructions of Thaddeus in [T]. Along the way we also get local and global Torelli-type theorems for the moduli spaces of stable pairs considered in [T].

More precisely, if P_ω denotes the moduli spaces of pairs of rank 2, degree d, and fixed determinant, stable with respect to the weight $\alpha \in (\max(0, (d/2) - \omega - 1), ((d/2) - \omega))$, we have the following.

THEOREM 1. *Let C be a smooth curve of genus $g \geqslant 3$. Then for all P_ω ($\omega \geqslant 1$), the Weil-Griffiths intermediate Jacobian $J^2(P_\omega)$ associated to the third cohomology $H^3(P_\omega, \mathbb{Z})$, is canonically isomorphic to the Jacobian, $J(C)$, of C. That is, we have an isomorphism*

$$J(C) \cong J^2(P_\omega)$$

of abelian varieties.

THEOREM 2. *If C is a smooth curve of genus $g \geqslant 4$ and if $\deg V = d \geqslant 2g - 2$ for $(V, s) \in P_\omega$, then we have*

$$h^i(P_\omega, \mathscr{T}_{P_\omega}) = \begin{cases} 4g - 3 & i = 1 \\ 0 & i = 0, 2. \end{cases}$$

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