

ON WHITTAKER COEFFICIENTS OF SOME METAPLECTIC FORMS

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At a conference in Montreal in 1989, Bump, Friedberg, and Hoffstein pronounced the conjecture that to a pair of modular forms f, g in one variable of half-integral weight, one could find an automorphic form on the double cover \widetilde{GSp}_2 of the symplectic group of rank 2, whose Whittaker coefficients are products of the Fourier coefficients of f and g [7]. At about the same time in [2], two of the present authors announced the result that averages of Fourier coefficients of certain holomorphic Siegel modular forms F of weight and degree 2 can indeed be expressed in a simple way by the products of the Fourier coefficients of a pair of modular forms of weight $3/2$ associated to them; more complete versions of this result have appeared since [3], [5], [6]. In this article we want to show that the last examples can be transformed into examples for the conjecture mentioned above by applying to F a theta lifting from $PGSp_2 \cong SO(3, 2)$ to \widetilde{GSp}_2 . The key idea is to use the computation of the Whittaker coefficients of such lifted forms on \widetilde{Sp}_2 from [13] and to express the result of that computation in terms of the classical Fourier coefficients of F .

Let $G_1 = Sp_2 \subseteq GL_4$ be the symplectic group of rank 2, $G = GSp_2 \cong G_1$ the group of symplectic similitudes, both groups viewed as algebraic groups over \mathbb{Q} . For $R = \mathbb{Q}_p$ (p a prime or ∞) or $R = \mathbb{A}$ (the ring of adèles of \mathbb{Q}) we let $\widetilde{G}(R)$, resp. $\widetilde{G}_1(R)$, be the twofold metaplectic cover of $G(R)$, resp. $G_1(R)$. These covers split over the maximal unipotent subgroup

$$U = \left\{ u = u_{a,b} = \begin{pmatrix} 1 & a & * & * \\ & 1 & * & b \\ & & 1 & 0 \\ & & -a & 1 \end{pmatrix} \right\}$$

of G_1 .

We fix the standard additive character ψ of \mathbb{A}/\mathbb{Q} and derive from it the character (again denoted by ψ)

$$\psi(u_{a,b}) = \psi\left(a + \frac{1}{2}b\right)$$

of $U(\mathbb{A})$.

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