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COEFFICIENT ESTIMATES ON WEIGHTED BERGMAN SPACES

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Section 0. Introduction. Let A denote normalized area measure for the unit disk $\mathbb{D}$ in $\mathbb{C}$. The *Bergman space* L_a^2 is the subspace of the Hilbert space $L^2(A)$ consisting of functions that are also analytic in $\mathbb{D}$. The monomials z^n are orthogonal in L_a^2, and have norm $1/\sqrt{n+1}$; so a holomorphic function $f(z) = \sum_{n=0}^{\infty} a_n z^n$ is in L_a^2 if and only if $\sum_{n=0}^{\infty} |a_n|^2(1/(n+1)) < \infty$, and if this is so, the partial sums $\sum_{n=0}^{N} a_n z^n$ are polynomials converging to f. The weighted Bergman spaces normally studied are obtained by replacing the measure $dA(z)$ by the radial measure $dA_\alpha(z) = (1-|z|^2)^\alpha\, dA(z)$; in these spaces the monomials are again orthogonal, and a function is approximable in norm by the partial sums of its power series at zero. In this paper we are interested in studying nonradial weights of the form $|m(z)|^2\, dA_\alpha(z)$, where m is the modulus of a function in H^∞, the space of bounded analytic functions on $\mathbb{D}$.

There are two different ways of generalizing the Bergman space to these weights. We shall use $L_a^2(|m|^2 A_\alpha)$ to denote the space of analytic functions on $\mathbb{D}$ that also lie in $L^2(|m|^2 A_\alpha)$, and $P^2(|m|^2 A_\alpha)$ to denote the closure of the polynomials in $L^2(|m|^2 A_\alpha)$. These two spaces are, in general, different. If m has no zeroes, $L^2(|m|^2 A_\alpha)$ is just the set of quotients $\{f/m : f \in L_a^2\}$; but this coincides with $P^2(|m|^2 A_\alpha)$ only when $1/m$ is in $P^2(|m|^2 A_\alpha)$, which in turn is equivalent to requiring that 1 lie in the L_a^2 closure of $\{mp: p \text{ a polynomial}\}$, i.e., that m be a cyclic vector for multiplication by z (this is discussed in Section 3 below). When m has an infinite number of zeroes, we do not know when $P^2(|m|^2 A_\alpha)$ is all of $L_a^2(|m|^2 A_\alpha)$.

We are interested in obtaining, for functions in $P^2(|m|^2 A_\alpha)$ and $L_a^2(|m|^2 A_\alpha)$, estimates on the size of the Taylor coefficients, and on the rate of growth as the boundary is approached. Our principal results are the following.

THEOREMS 2.1, 2.6, 1.4. *Let m be in H^∞. Let f be in $L_a^2(|m|^2 A_\alpha)$. Then*

$$|\hat{f}(n)| = e^{O(\sqrt{n}\log n)}.$$

If f is in $L_a^2(|m|^2 A)$, then

$$|\hat{f}(n)| = e^{O(\sqrt{n})}.$$

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