UNIQUENESS OF WHITTAKER FUNCTIONALS ON THE METAPLECTIC GROUP

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0. Introduction. Let G be a finite central extension of a reductive split connected group over a locally compact, totally disconnected field F. By abuse of notation, we now also write G for the group of all F-rational points of G. Let (π, V) be any representation of G, and denote by N and Z the unipotent radical of G, and an abelian subgroup of G containing the center of G, respectively. Fix characters $\psi: N \to \mathbb{C}$, and $\mu: Z \to C$. Then a (μ, ψ)-Whittaker functional l is a linear functional $l: V \to \mathbb{C}$ such that

$$(0.1) \qquad\qquad l(\pi(zn)v) = \mu(z)\psi(n)l(v)$$

for all $z \in Z$, $n \in N$. In general, one hopes that with sufficient restrictions on G or (π, V), one can obtain theorems concerning the existence and uniqueness (up to scalar multiples, of course) of such a functional. Uniqueness of Whittaker functionals and Whittaker models is a problem which has been studied (and applied to the theory of automorphic forms) by Jacquet-Langlands, Gelfand-Graev, Gelfand-Kazhdan, Piatetski-Shapiro, Shalika, and Rodier when G is an algebraic group, and by Gelbart-Howe-Piatetski-Shapiro, Deligne, and Kazhdan-Patterson in the case when G is a metaplectic cover. Uniqueness principles have emerged as a critical subject in the theory of automorphic forms. Among applications of the uniqueness of Whittaker functionals, one finds Eulerian Rankin-Selberg integrals, Eulerian Whittaker-Fourier coefficients of Eisenstein series, and the normalization and analytic continuation of intertwining operators.

For the case of the double cover of $GL(2)$, Gelbert, Howe, and Piatetski-Shapiro [GHP-S] have shown that uniqueness always holds, and that every infinite-dimensional representation admits at least one character μ such that a functional satisfying (0.1) exists.

For higher covers, or for covers of higher-rank groups, it is not possible to prove even uniqueness. Indeed, even on the 3-fold cover of $GL(3)$, uniqueness no longer holds for arbitrary representations. By considering specific representations, however, one is still able to obtain results concerning the uniqueness of the Whittaker functional. For example, the generalized theta series of Kazhdan and Patterson [KP] have unique Whittaker functionals on the r-fold cover of $GL(n)$ when $n = r$ or $n = r - 1$.