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ON THE CAUCHY AND INVARIANT MEASURE PROBLEM FOR THE PERIODIC ZAKHAROV SYSTEM

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0. Introduction. The purpose of this preliminary paper is to prove local and global existence and regularity theorems for the 1-dimensional Zakharov model

$$\begin{cases} iu_t = -u_{xx} + nu \\ n_{tt} - n_{xx} = (|u|^2)_{xx} \\ u(x, 0) = \varphi, \quad n(x, 0) = a, \quad \partial_t n(x, 0) = b \end{cases} \tag{0.1}$$

in the space periodic case. Apparently no results are known so far on this problem, if one considers the periodic setting (*). Equations (0.1) are suspected to be nonintegrable, contrary to the nonlinear Schrödinger equation (NLSE) $iu_t + u_{xx} + u|u|^2 = 0$. The technique used here is a Fourier analysis approach in the same spirit as earlier works in [B1], [B2] on NLSE and KdV type equations. In particular we prove following local wellposedness theorem.

THEOREM 1. *There are Sobolev exponents $0 < \sigma < s < 1/2 < s_1 < 1$ such that (0.1) is locally wellposed for data (u, a, b) satisfying*

$$\varphi \in H^s(\mathbb{T}), \qquad \sup_k |k|^{s_1} |\hat{\varphi}(k)| < \infty$$

$$\sup_k |k|^{-\sigma} |\hat{a}(k)| < \infty \qquad \textit{and} \qquad \sup_k |k|^{-\sigma-1} |\hat{b}(k)| < \infty .$$

Here one should think of σ close to 0, s close to 1/2. In particular, for (H^1, L^2, H^{-1})-data, there is the following global result.

THEOREM 2. *The system (0.1) is globally wellposed for data $\varphi \in H^1$, $a \in L^2$, $b \in H^{-1}$.*

Due to the conservation of the Hamiltonian

$$H_Z = \frac{1}{2} \int_{\mathbb{T}} \left[|u_x|^2 + \frac{1}{2}(n^2 + V^2) + n|u|^2 \right] dx \tag{0.2}$$

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(*) There have been a number of investigations on this issue in the nonperiodic case, starting from the paper [SS] in 1-dimension.