

# BOTT TOWERS, COMPLETE INTEGRABILITY, AND THE EXTENDED CHARACTER OF REPRESENTATIONS

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**Introduction.** Two themes are interwoven throughout this paper, representation theory and symplectic geometry. These two are connected via the “orbit

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