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TOPOLOGICAL RADON TRANSFORMS AND THE LOCAL EULER OBSTRUCTION

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1. Introduction. The local Euler obstruction was introduced independently by MacPherson and Kashiwara in the beginning of the 1970's. Actually it was not until later that Dubson [4] discovered that the local Euler obstruction defined by MacPherson and the local characteristic defined by Kashiwara are the same thing.

MacPherson [26, page 425] used the local Euler obstruction in his celebrated work, establishing a conjecture of Grothendieck and Deligne. MacPherson proved that the local Euler obstruction is a constructible function and gives rise to an isomorphism from the group of algebraic cycles $Z(X)$ to that of constructible functions $F(X)$. The MacPherson-Chern class is the inverse of this isomorphism composed with the morphism from the group of algebraic cycles to the Chow group given by the Mather-Chern class. Furthermore, MacPherson constructed a push-forward for algebraic cycles under proper morphism using a graph-construction, such that the local Euler obstruction and the Mather-Chern class are functorial with respect to proper push-forward.

Independently, at about the same time, Kashiwara [14, page 804] proved an index theorem for holonomic D-modules involving the local Euler obstruction of certain characteristic cycles associated to the given D-module.

In 1978, Verdier and Gonzalez-Sprinberg gave an equivalent algebraic definition of the local Euler obstruction [12, page 15]. Consider the Nash blow-up of X with its projection to X. The local Euler obstruction at a point $x \in X$ is equal to the degree of the total Chern class of the tautological bundle of the Nash blow-up, acting on the Segre class of the fiber over x in the Nash blow-up.

In 1985, Sabbah [32, page 166] and Kennedy [16, pages 2827–2830] reformulated MacPherson's theory, replacing algebraic cycles with Lagrangian cycles on the cotangent space of a nonsingular ambient variety. The push-forward of Lagrangian cycles was constructed using Fulton's and MacPherson's [9], [8] algebraic intersection product. The deformation to the normal cone corresponds here to the deformation on a Grassmanian used in the MacPherson graph-construction.

In this paper we will apply the theory of MacPherson-Chern classes to study the singularities of projective varieties. It is possible to define what are called topological Radon transforms of constructible functions on $\mathbb{P}^N$. In fact, in 1986 Brylinski [1, pages 24–62] defined the Radon transform for constructible sheaves. However, here we will restrict our attention to constructible functions. Fix an

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