Vol. 75, No. 1 DUKE MATHEMATICAL JOURNAL © July 1994

UNITARY REPRESENTATIONS OF BRIESKORN SPHERES

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1. Introduction. There is a rich and elegant theory of representations of finite groups. Up to conjugation, there are only finitely many distinct irreducible representations in any given rank, and the collection of all irreducible representations satisfy a famous arithmetic relation [17].

Suppose that (p, q, r) are pairwise relatively prime, and let $\Sigma(p, q, r)$ be a Brieskorn sphere, that is, the link of the singularity of the variety $x^p + y^q + z^r = 0$ in $\mathbb{C}^3$. The rank-two representation theory of the groups $\pi_1 \Sigma(p, q, r)$ shares many properties with that of finite groups. In particular, up to conjugation, there are only finitely many irreducible representations of rank two. Counting the number of these representations immediately yields Casson's invariant. This follows from the observation of Fintushel and Stern [8] that the SU(2) spectral flow of any Seifert fibered homology sphere is even and the characterization of Casson's invariant as the Euler characteristic for Floer homology [18]. In the general case of a Seifert fibered homology sphere $\Sigma(a_1, \ldots, a_n)$ (the link of the singularity of complete intersection of complex dimension 2 in $\mathbb{C}^n$), the SU(2) representation space is not discrete but has components of dimension $2m$ for each $0 \leqslant m \leqslant n - 3$. The perturbation argument of §4 of [8] along with the results in [12] show that Casson's invariant of $\Sigma(a_1, \ldots, a_n)$ is just the Euler characteristic of its SU(2) representation space.

In this paper, we consider the problem of describing in rather general terms the SU(N) representation space of Seifert fibered homology spheres $\Sigma(a_1, \ldots, a_n)$. For example, it is shown that the odd-dimensional homology of any connected component of irreducible SU(N) representations vanishes. This is done by interpreting it as the moduli of parabolic bundles over the Riemann sphere. Since any component of irreducible SU(3) representations of a Brieskorn sphere has dimension $\leqslant 2$, it follows that it is either a point or a two sphere. Restricting our attention further to the Brieskorn spheres $\Sigma(2, p, q)$, the SU(3) representation space is just a discrete set of points.

By the *leading term* of the SU(3) Casson invariant, we mean $\sum_\rho (-1)^{SF(\Theta, \rho)}$ where the sum is taken over ρ an irreducible representation in a (possibly perturbed) representation space. The other term is a sum of Maslov indices over the reducibles, which is more subtle to define and is not discussed here (cf. [7]). For ρ an irreducible SU(N) representation of a Seifert fibered homology sphere, we prove that $SF(\Theta, \rho)$ is always even. Thus, for the Brieskorn spheres $\Sigma(2, p, q)$, the calculation of the leading term of the SU(3) Casson invariant is reduced to the

Received 21 January 1994.