Vol. 74, No. 3 DUKE MATHEMATICAL JOURNAL © June 1994

A NONVANISHING RESULT FOR TWISTS OF L-FUNCTIONS OF GL(n)

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In this article we prove the following.

THEOREM. *Let F be a number field, n an integer $\geqslant 3$, and $T = [1/n, 1 - (1/n)] \subset \mathbf{R}$. Let π be a unitary cuspidal automorphic representation of $\mathrm{GL}_n(\mathbf{A}_F)$, S a finite set of places of F, and s_0 a complex number such that $\Re s_0 \notin T$. Then there exist infinitely many primitive ray class characters χ of F such that χ is unramified at the places of S and*

$$L(\pi \otimes \chi, s_0) \neq 0.$$

Moreover, suppose π is tempered, i.e., it satisfies the generalized Ramanujan conjecture. Then the same result holds with T replaced by $T_1 = [2/(n + 1), 1 - (2/(n + 1))]$.

Such a result was established by David Rohrlich in [14] for GL(1) and GL(2) at every point s_0 in $\mathbf{C}$, i.e., with the exceptional set T being empty. It may be worthwhile to note that our result here gives a nonvanishing statement for twists of the L-functions of cuspidal tempered automorphic representations of $\mathrm{GL}_3(\mathbf{A}_F)$ at every point s_0 outside the critical line $\Re(s) = 1/2$.

The case $\Re s > 1$ is trivial since the L-function has a convergent Euler product expansion in this region. It is also well known that $L(\pi, 1 + it) \neq 0$, but we do not make use of this in order to stress that the method used here works for $\Re s = 1$ as well. We hope that the method will give analogous results for other groups. In fact the original motivation for this work came from our attempt to prove a nonvanishing result for the degree-5 L-functions of GSp(4) at $s = 1$, which if established will have implications for the classification of automorphic representations of GSp(4). We hope to treat this case in a future work.

Our proof follows the method used by Rohrlich, which consists in proving that for a large enough product q of distinct primes in $\mathbf{Z}$, the average value of $\{L(s_0, \pi \otimes \chi) | \chi$: primitive finite-order character of conductor $\mathfrak{q}$ of norm $q\}$ is nonzero. (We have tried to use notations consistent with his.) However, we need some additional inputs; they are: bounds for certain Kloosterman-type sums due to Deligne [5], the behaviour of root numbers under twisting, the properties of L-functions of GL(n) and GL(n) × GL(n), and most importantly, a finer (s_0-dependent) estimate of the crucial sum Σ_{22} of [14] (see Proposition 5.1 below). We give full details for the parts which require arguments beyond the case $n = 2$, referring

Received 6 October 1993. Revision received 30 November 1993.