Vol. 74, No. 3 DUKE MATHEMATICAL JOURNAL © June 1994

NONEXISTENCE RESULTS FOR SOLUTIONS OF SEMILINEAR ELLIPTIC EQUATIONS

RAFAEL D. BENGURIA, SEBASTIÁN LORCA, AND CECILIA S. YARUR

1. Introduction. In the last few years there have been a number of articles dealing with existence and nonexistence results for semilinear elliptic equations in $\mathbb{R}^N$. This type of equation comes from many sources, mainly from geometry and physics (see, e.g., [3], [4], [5], [6], [7], [12], [14], [15], [17], [22]). Here we will only be concerned with criteria for the nonexistence of nontrivial solutions of semilinear elliptic partial differential equations. On the existence problem there is a vast literature. We refer the reader to [23] for a survey on recent developments and a brief history. Our purpose is to give a rather unified treatment of the problem of nonexistence of solutions for the equation

$$-\Delta u + Vu + f(x, u) = 0, \qquad \text{in } \mathbb{R}^N, N \geqslant 3, \tag{1.1}$$

where $uf(x, u) \geqslant 0$, including the case where $V(x)$ is not necessarily positive. In particular we will extend the nonexistence results of Ni [22] and Lin [18]. The basic strategy consists of reducing the partial differential equation (PDE) problem to the study of the solutions of an associated ordinary differential equation (ODE) using convexity results.

Our main results are the following.

THEOREM 3.3. *Let $u \geqslant 0$ be a solution of*

$$-\Delta u + K(x)u^q = 0 \qquad \text{in } \mathbb{R}^N, N \geqslant 3, \tag{1.2}$$

where $q > 1$, $K(x) \geqslant 0$ in $L^\infty_{loc}(\mathbb{R}^N)$ and $K_q(r) \geqslant \phi(r)$ at infinity (K_q given by (2.15) below), where $\phi \in C^1$ satisfies:

(i) $\int^\infty r\phi(r)\, dr = +\infty$, *and*

(ii) $(\phi(r)r^{2(N-1)})' \geqslant 0$, *near infinity.*

Then

$$u \equiv 0.$$

Remarks. (i) Theorem 3.3 above is close to being optimal, since in [21] Naito proves that if $|K(x)| \leqslant \phi(|x|)$ for all $x \in \mathbb{R}^n$, and $\int_0^\infty r|\phi(r)|\, dr < \infty$, then (3.3) has positive solutions.

Received 1 December 1992. Revision received 12 November 1993.