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SQUARE ROOT FORMULAS FOR CENTRAL VALUES OF HECKE L-SERIES II

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1. Introduction. This paper is a complement to [8]; we show how the results proved there can be extended to quadratic imaginary fields K with arbitrary odd discriminant $-d < -3$, hence fulfilling, in part, the promise made in [7]. We consider the central value of the L-series $L(\psi, s)$ associated to a Hecke character ψ of K that satisfies (see (1))

$$\psi((\alpha)) = \varepsilon(\alpha)\alpha^{2k-1}, \qquad \text{for integers } \alpha \text{ of } K \text{ prime to } d.$$

We show that one can associate to every such ψ a well-defined genus $\mathscr{G}_\psi$ and that the central value $L(\psi, k)$ equals $c|\mathscr{T}|^2$, where c is a simple normalizing factor and $\mathscr{T}$ is a linear combination of values of the $(k-1)$st nonholomorphic derivative of a classical half-integral weight theta series, at CM points corresponding to ideals in $\mathscr{G}_\psi$. This is our main result (14).

The essential new feature is the analysis of the genus theory involved. For nonprime discriminants the class number of K is even, and hence we cannot rely on having squares of ideals in every class as it happened in [8] and [7].

We have organized the paper as follows: a main part and an appendix. We also included a list of minor corrections to [8]. In the main part, we set things up in Section 2, deal with genus theory in Section 3, put together the final formula in Section 4, after recalling the key ingredients of [8], and present some numerical examples in Section 5. In the appendix we briefly discuss some ideas related to the factorization formula (12). This sheds some light on how such formulas work; for example, we show how in general they are truly identities of values holding only on a finite number of points. It also gives an indication of more general formulas, which we will discuss elsewhere.

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2. Basic setup. Let K be an imaginary quadratic field of discriminant $-d$, with $d > 3$, $d \equiv 3 \bmod 4$, viewed as a subfield of the complex numbers $\mathbf{C}$. We understand by $\sqrt{-d}$ the root with positive imaginary part.

Let $\mathcal{O}_K$ be the ring of integers of K; note that $\mathcal{O}_K^* = \{\pm 1\}$. Unless stated otherwise, by ideals we will always mean integral ideals. An ideal is primitive if it is not divisible by rational integers > 1. We let Cl be the class group of K, and $Cl_{(2)}$ the subgroup

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