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ON CONFORMAL DEFORMATION OF NONPOSITIVE CURVATURE ON NONCOMPACT SURFACES

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0. Introduction. Let (X, g) be a 2-dimensional Riemannian manifold and let K be a given function on X. The question of conformal deformation of Gaussian curvature (or prescribing Gaussian curvature) is to find a conformal metric of the form $g_1 = e^u g$ such that K is the Gaussian curvature of g_1. This is equivalent to the problem of solving the elliptic equation

$$\Delta u - 2k + 2Ke^u = 0 \tag{0.1}$$

on (X, g), where Δ and k are the Laplacian and the Gaussian curvature on (X, g). This problem has been considered by many authors in both compact and noncompact cases. We refer the reader to [7] for references for the compact case. If (X, g) is a noncompact surface, it follows from the uniformization theorem that every noncompact Riemannian surface is conformal to a complete Riemannian surface of constant Gaussian curvature $k = 0$ or $k = -1$. In the case that $k = 0$ and (X, g) is conformal to the flat Euclidean plane $\mathbf{R}^2$, this problem has been studied extensively in [1], [3], [8], [10], [11], [12], [15], etc. If $k = -1$, then the equation (0.1) is reduced to

$$\Delta u + 2 + 2Ke^u = 0. \tag{0.2}$$

The model case for $k = -1$ is the hyperbolic disc $\mathbf{H}^2$, which was studied by J. Bland and M. Kalka in [2] under the conditions that K is a nonpositive function which is bounded both from above and below by negative constants near ∞. Their results have recently been generalized by D. Hulin and M. Troyanov in [6] to the case that (X, g) is a noncompact Riemannian surface of finite topological type.

The aim of this paper is to study the conformal deformation of nonpositive Gaussian curvature on noncompact Riemannian surfaces. We are able to treat this problem for general Riemannian surfaces without any topological restrictions. For the existence results, the upper bound on K by a negative constant in [2] and [6] is replaced by a very weak negativeness condition. We are also able to determine precisely how fast K can grow to $-\infty$ while still insuring that equation (0.2) has a

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