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SCATTERING THEORY FOR SYMMETRIC SPACES OF NONCOMPACT TYPE

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1. Introduction. This paper arose out of an attempt to understand the seminal work of M. A. Semenov-Tian-Shansky [STS] on harmonic analysis for Riemannian symmetric spaces of negative curvature. His theory provides a beautiful generalization of the Lax-Phillips scattering theory [LP], which in this context only applies to Lie groups of real rank one. A subsequent paper by M. Shahshahani [S1] on this subject gives a more direct approach and clarifies the setting for the problem. The present paper leans heavily on both of the above papers. However, by going directly to the various translation representations we are able to come up with a simpler and more coherent development.

Semenov-Tian-Shansky devised a hyperbolic system of equations, one for each time direction, whose solution is given by a commutative family of unitary operators on a suitable Hilbert space. What is novel about this problem is the appearance of a multiple time frame (of dimension equal to the rank of the symmetric space) and the profusion of the translation representations (one for each element of the Weyl group).

Our starting point is to expand the Fourier transform of the solution of the hyperbolic system in terms of the eigenvectors of the generators. Although the various components are no longer the Fourier transforms of data, they each provide a different translation representation, and their individual inverse Fourier transforms, surprisingly, yield the solution to the original problem. The resulting theory is more in the spirit of Shahshahani than Semenov-Tian-Shansky. It allows us to simplify and in some cases sharpen many of the earlier results.

A review of the background material needed in the subsequent development is given in Section 2. In Section 3 we describe the system of differential equations and the energy form introduced by Semenov-Tian-Shansky. We then show that the various components of the Fourier transform of the solution all carry the same energy and that the solution to the original problem can be recovered from each of them. By taking the Euclidean Fourier transform of these components we obtain the various translation representations.

Finally in Section 4 we use the different translation representations to study the associated outgoing subspaces. To begin with, the σ-outgoing subspace D_σ is defined

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