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CONSTRUCTING FREE ACTIONS ON R-TREES

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Introduction and statement of results. This paper describes two constructions leading to free group actions on **R**-trees. In the first one we start with an arbitrary action (G, T), and we construct actions of certain quotients G/H on quotient **R**-trees $\widehat{T/H}$. Among these actions, there is a "largest" free one, so that we can associate a free action to (G, T) in a canonical way. In the second construction, we use pseudogroups of rotations of the circle constructed in [Le3] to get free nonsimplicial actions of the free group of rank 3. The translation lengths of the generators may be any triple of positive, rationally independent numbers. Both constructions use measured foliations.

To introduce quotient actions, let G be a group acting isometrically on a metric space (X, d), and $H \subset G$ a normal subgroup. Consider X/H, the set of orbits of the restriction of the action to H. The metric of X induces a pseudodistance on X/H, given by $d_H(Hx, Hy) = \inf_{h,h' \in H} d(hx, h'y)$, and we let $\widehat{X/H}$ be the associated metric space. Obviously, G/H acts isometrically on $\widehat{X/H}$.

In order to apply this to **R**-trees, we shall determine when $\widehat{X/H}$ is an **R**-tree, assuming that X is an **R**-tree.

Let therefore H be a group acting on an **R**-tree T. Let $\ell\colon H \to \mathbf{R}^+$ be the associated length function $\ell(h) = \inf_{x \in T} d(x, hx)$. Recall that this infimum is always achieved; in particular, $\ell(h) = 0$ if and only if h acts with a fixed point. (We then say h is *elliptic*.) See the surveys [Sh1], [Sh2], [Mo] for basic facts about **R**-trees.

Given $c \in \mathbf{R}$, with $0 < c \leqslant 1/3$, say that the action of H (or the length function ℓ) satisfies condition $(*)$ if the following holds: given $h \in H$ and $\varepsilon > 0$, one can write $h = h_1 h_2$ with $h_1, h_2 \in H$ and

$$\begin{cases} \ell(h_1) + \ell(h_2) < \ell(h) + \varepsilon \\ \max(\ell(h_1), \ell(h_2)) < (1 - c)\ell(h) + \varepsilon. \end{cases} \tag{$*$}$$

THEOREM 1. *Let H be a countable group acting on an* **R**-*tree T, with length function ℓ. Then $\widehat{T/H}$ is an* **R**-*tree if and only if ℓ satisfies* $(*)$.

Remarks. The choice of c in $(0, \frac{1}{3}]$ is irrelevant, but the theorem would be false with $c > \frac{1}{3}$ (see Example III.2). If $\widehat{T/H}$ is isometric to a subinterval of **R**, then ℓ satisfies $(*)$ with $c = \frac{1}{2}$ (see Remark III.1).

If $\ell = |\tau|$, where $\tau\colon H \to \mathbf{R}$ is a homomorphism, then ℓ satisfies $(*)$ if and only if $\tau(H) \not\simeq \mathbf{Z}$.

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