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ISOSPECTRAL COMPACT FLAT MANIFOLDS

ISABEL DOTTI MIATELLO AND ROBERTO J. MIATELLO

1. Introduction. It was shown by Milnor [4] that there exist pairs of isospectral, nonisometric flat tori of dimension 16. (It is now known that such tori exist in dimension $\geqslant 8$ [3].) Also, each isospectral class of compact flat Riemannian manifolds contains finitely many isometry classes [8].

The purpose of the present paper is to show that isospectrality is quite a common phenomenon among compact flat manifolds, even in low dimensions, and with zero first Betti number (see Corollary 3.2). Our main result is the following theorem.

THEOREM 1.1. *If $n \geqslant 5$, there exist pairs of isospectral nonhomeomorphic compact flat Riemannian manifolds M, $\overline{M}$ of dimension n, with holonomy group Z_2^k, $2 \leqslant k \leqslant n-3$. Furthermore, M, $\overline{M}$ can be chosen so that $\beta_1 = h$ for any h with $1 \leqslant h \leqslant n-3$, $i = 1, 2$. If $n \geqslant 6$, then we can choose M, $\overline{M}$ to have $\beta_1 = 0$, $i = 1, 2$ and holonomy group Z_2^k for any k with $3 \leqslant k \leqslant n-3$.*

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2. Certain Bieberbach groups. A discrete cocompact subgroup Γ of $I(\mathbf{R}^n)$ is said to be a crystallographic group. A Bieberbach group is a crystallographic group which is torsion-free. A compact connected flat Riemannian manifold M has euclidean space $\mathbf{R}^n$ as its universal covering space and a Bieberbach group Γ as fundamental group. By Bieberbach's first theorem, if Λ denotes the subgroup of translations in a crystallographic group, then Λ is a lattice in $\mathbf{R}^n$. Furthermore, Λ is a normal and maximal abelian subgroup of Γ. When Γ is torsion-free, the geometric interpretation of $\Lambda \backslash \Gamma$ is that of the (linear) holonomy group of the flat manifold M.

By a theorem of Auslander-Kuranishi [1], any finite group G arises as the holonomy group of a compact flat manifold. On the other hand, it is an open question, in general, to find the minimal dimension for flat manifolds with prescribed G as its holonomy group (see for instance [5]). The purpose of this section is to describe a concrete construction which allows one to produce examples in dimension $n \geqslant 3$, of compact flat manifolds having holonomy group $\mathbf{Z}_2^k$, $1 \leqslant k \leqslant n-1$.

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