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ANALYTICITY PROPERTIES IN SCATTERING AND SPECTRAL THEORY FOR SCHRÖDINGER OPERATORS WITH LONG-RANGE RADIAL POTENTIALS

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1. Introduction. In this paper we consider Schrödinger operators of the form

$$P = -\Delta + V \qquad \text{in } \mathbf{R}^n, \tag{1.1}$$

where V is a complex radial potential of long range. We are interested in the relation between the resolvent kernel (the Green's function), the generalized eigenfunctions, and the scattering matrix, and we shall study the problem of obtaining an analytic continuation of all these objects in the "energy" parameter.

In contrast to the case of a short-range potential, even the notion of generalized eigenfunctions and the scattering matrix needs a clarifying definition for a long-range potential; the Green's function, however, is uniquely determined. Our definition, in the framework of time-independent scattering theory, will involve the asymptotic behavior of the Green's function at infinity. To describe this asymptotic behavior we shall choose some appropriate phase function (for which there is a canonical choice in the short-range situation); each possible choice will then define a family of closely related generalized eigenfunctions and a scattering matrix. We remark that this construction is simplified considerably for the case of a radial potential which we consider in this paper.

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